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Application of particle swarms for project time-cost optimization

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Preliminary note

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The time-cost optimization of a construction project, as related to the costs of its activities and the total project realization costs, is considered in this paper. Direct and indirect costs, durations of all activities, project realization time, and relations between activities, are taken into account. The particle swarm optimisation method is applied for solving this nonlinear problem. The authors adjusted this method for the time-cost optimization, and developed for that purpose an appropriate computer programme in the scope of the MATLAB programming system.

Key words:
project management, project planning methods, time-cost optimization of projects, particle swarm optimisation

Prethodno priopćenje

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Primjena rojeva čestica za optimizaciju vremena i troškova projekta

U ovome radu se razmatra optimizacija vremena izvođenja građevinskog projekta u odnosu na troškove izvršenja njegovih aktivnosti i ukupne troškove izvođenja projekta. Uzimaju se u obzir direktni i indirektni troškovi, duljine trajanja svih aktivnosti i duljina izvođenja projekta, te veze između aktivnosti. Za rješavanje ovog nelinearnog problema primijenjena je metoda optimizacije pomoću čestica rojeva, koju su autori prilagodili za optimizaciju troškova i vremena i razvili odgovarajući računalni program u programskom sustavu MATLAB.

Ključne riječi:
projektni menadžment, metode planiranja projekata, optimizacija vremena i troškova projekta, optimizacija pomoću rojeva čestica

Vorherige Mitteilung

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Anwendung von Partikelschwärmen zur Optimierung von Projektdauer und Kosten

In dieser Arbeit wird die Optimierung der Ausführungsdauer und Kosten von Bauprojekten in Bezug auf die Abwicklungskosten der Aktivitäten und die gesamten Ausführungskosten des Projekts betrachtet. Direkte und indirekte Kosten, Ausführungsdauer und Zusammenhänge einzelner Tätigkeiten werden in Betracht gezogen. Um dieses nichtlineare Problem zu lösen, wird die Methode der Partikelschwarmoptimierung angewandt, die von den Autoren zur Optimierung von Kosten und Zeit angepasst und in der Form eines Computerprograms in das Softwaresystem MATLAB implementiert wurde.

Schlüsselwörter:
Projektmanagement, Projektplanungsmethoden, Optimierung von Projektdauer und Kosten, Partikelschwarmoptimierung

1. Introduction

Construction projects are target-oriented and planned undertakings, whose objective is to build, reconstruct or remodel various construction facilities. These projects involve dynamic processes that can be divided into four phases: conceptualisation, definition, realization, and use of the construction facility. Considerable funds are invested in the realization of these phases. The latter are characterized by participation of a considerable number of companies, institutions, and organisations, and by the use of considerable quantities of various resources and machinery. The realization of construction projects is often time-consuming, and contractors are required to complete the project within the contracted time, in accordance with high quality standards, and at lowest possible costs. This is why participants in construction are faced with the problem of how to optimise construction time, minimise construction costs, and respect other relevant criteria. The activities important for the realization of construction projects will be considered in this paper because, being related to the time, costs, work quantities, technologies used, and work processes, they actually take up the dominant part of construction projects. The project realization phase starts with conclusion of contract between the client and the contractor, and ends by the final inspection, handover of the project to the client, and by the delivery of the operating permit. The optimization method described in this paper can also be applied for realization of other phases of a construction project.

The start of development of methods for the planning and control of realization of projects and production is associated with the work of Henry L. Gantt, an American engineer and one of the pioneers of scientific management and organisation of work [1]. In 1917, Gantt proposed and developed a method that is still in use today, i.e. the Gantt chart method, where activities are presented in form of lines. First methods with mathematical formulations for construction scheduling, cost and time optimization, and cost or resource use levelling, were proposed in the 1950s [2-4]. The American corporation E.I. du Pont de Nemours & Company was formed a team in 1955 in order to improve and develop new planning techniques. This team proposed a new planning technique named the *Critical Path Method (CPM)* with the critical path diagram in which activities are presented in form of arrows, while events are presented as circles. First papers for determining an optimum work scheduling using the primal and dual linear programming problem were published in 1961 [2, 3]. It is at that time that the Fondahl's heuristic method for project cost optimization, based on linear dependence between the activity costs and activity realization time, was also developed [5]. In this method, called the "Precedence Method" (PMD method), the critical path diagram contains activities presented in nodes and marked with circles or rectangles, while their correlations and orders of precedence are marked with arrows. This heuristic optimization method was modified, simplified and adjusted to the realization of construction projects [6, 7]. The paper [8] analyses the cost of construction of several types of construction facilities/structures and proposes simple formulas for an approximate calculation of an economical cost-dependent construction time, while also proposing forms for defining the most favourable work front. The problem of a line of balance optimization is mathematically formulated in [9] as a linear programming task, taking into account direct and indirect construction costs to which this kind of plans can be applied. The heuristic approach is used to optimise the critical path diagram involving a great number of activities, and the results are obtained using the project management program package PCS (Project Control System) [10]. Optimization problems in construction industry and optimization of industrial facilities are considered in [11-17].

The single criterion problem of optimization of time as related to costs (time-cost trade-off) is considered in many professional and research papers. The single criterion procedure for determining an optimum time for realization of projects has some limitations, i.e. it does not fully consider the quantity of available resources, which are limited [18]. It is therefore indispensable to take into account available resources during determination of typical durations of activities, especially for a greater number of activities that take place at the same time. A considerable number of published papers deal with the issue of *levelling of resources*. The number of restrictions in the mathematical optimization model must be increased to take into account these factors, which greatly complicates determination of optimum solutions.

The complex problem of time and cost optimization under the influence of risks and limited resources is considered in [19]. The problem of resource control planning, construction time optimization, and distribution of resources for construction projects, is dealt with in [20, 21].

Heuristic methods, mathematical programming methods, and simulation methods, are used in the formulation of models and for solving optimization problems. Heuristic methods, including the Fondahl's method, are often considered unsuitable for solving optimization problems, especially for critical path diagrams with a great number of activities, as they require many steps for obtaining the solution. This approach, defined as non-computational by Fondahl, is unsuitable for the development of computer programs. The plan optimization problem solving using numerical and analytical mathematical programming methods, which are based on Karush-Kuhn-Tucker optimization conditions, is highly impractical due to a large number of unknown variables and constraints. In order to enable a more efficient solution of optimization problems, which are formulated as mathematical models, the use is increasingly made of genetic algorithms, evolutive strategies, simulated annealing techniques, particle swarms, ant colonies, etc. Probability methods and methods based on the theory of fuzzy sets are also used due to imprecision and uncertainty in the determination of parameters and model variables. The hybrid approach, which combines simulation techniques and genetic algorithms for solving time-cost optimization problems, is applied in [22]. It has been shown that this combination enables definition of optimum durations and optimum orders of activities. A more recent approach for

simultaneous optimization of the total project realization time and costs using genetic algorithms is presented in [23]. The time-cost optimization problem has also been solved using the ant colony method [24], genetic algorithm method, and Monte Carlo method [25-27].
In addition, the time-cost optimization problem has been solved using the stochastic linear programming, taking into account the variability of financing, and the uncertainty of project realization time [28]. The financial probability for realizing the project has been expressed as a stochastic constraint, while the uncertainty of realization is included in the model using the PERT method. The algorithm for determining the minimum project duration with limited resources is used in [29]. A hybrid evolutive algorithm for optimising the time and cost of realization of construction projects is developed in [30]. The multicriteria linear programming for an optimum planning of projects characterised by reiterating segments, as in line of balance planning, is used in [31]. The criteria are: project realization time, duration of individual segments, delay in realization of such segments, and total project costs. An optimum line of balance planning with resource limitations and an optimum assignment of workers to appropriate tasks using the genetic algorithm method, is presented in [32]. The particle swarm optimization method for bi-criterial time-cost analysis is applied in [33]. Direct costs and project realization time are taken as criteria, while constraints are formulated according to duration of activities and their mutual relationships. The proposed particle swarm algorithm is used to determine the Pareto front representing the curve that defines an optimum correlation between two chosen criteria. The problem of evaluation of total cost in building construction is considered in [34]. The problem of discrete time-cost optimization, with multimodal constraints as related to resources, and based on fuzzy genetic algorithms, is considered in [35-37].
This paper provides a mathematical formulation of the time-cost optimization problem as related to realization of construction projects, where the problem is solved by the particle swarm optimization, which is a recent optimization method. This algorithm, normally used for determination of optimum solutions in different areas, is adapted in this case to the solution of the time-cost optimization problems.

2. Mathematical optimization model

The mathematical model for the time-cost optimization during realization of construction projects, as presented in this paper, is formulated as a nonlinear (quadratic) programming problem with linear constraints. The mathematical model, as a problem of linear programming with the linear objective function and the linear constraints, was formulated in the early 1960s [2, 3].

2.1. Project realization time and constraints

As the mathematical model is formulated on the critical path diagram, the constraints arise from the mutual relationships between activities in the diagram and their possible durations, and the total realization of the project.
The minimum and maximum activity realization time can be calculated for each activity on the project A_i (i=1, 2, ...n_a). The minimum time TC_i (crash time) is the time during which this activity can be realized the fastest under given conditions, using a particular technology and available resources. In addition to these times, the so called conventional time TE_i is introduced [38], and this time is estimated for normal operating conditions.
In a general case, the relation between some given times can be shown using the following inequation:

$$TC_i \leq t_i \leq TN_i, \quad i = 1, 2, \dots n_a \tag{1}$$

where n_a is the number of activities on a project.

The following relation is valid for duration t_i of the activity A_i

$$TC_i \leq t_i \leq TN_i, \quad i = 1, 2, \dots n_a$$

or

$$t_i \geq TC, \; t_i \leq TN_i, \quad i = 1, 2, \dots n_a \tag{2}$$

If TC denotes the shortest time and TN the longest time, and t_{pr} the possible time of realization of a project, then the following is valid

$$TC \leq t_{pr} \leq TN$$

or

$$t_{pr} \geq TC, \quad t_{pr} \leq TN \tag{3}$$

The shortest project realization time TC is obtained for the shortest times TC_i of realization of activities A_i, while the longest project realization time TN is obtained for the longest times TN_i of realization of activities A_i (i = 1,2,...,n_a).
The length of the project realization phase t_{pr} is equal to the sum of duration of activities t_k on one of critical paths.

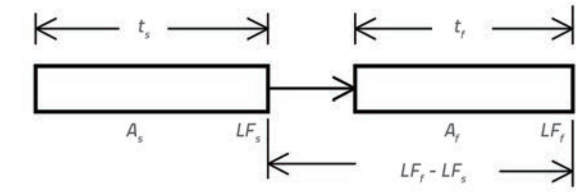
$$t_{pr} = \sum_k t_k \tag{4}$$

In practice, there is another important condition that arises from the contract concluded between the client and the contractor: the project realization phase must be completed within the contract time t_{ug}. Thus we have:

$$t_{pr} \leq t_{ug} \tag{5}$$

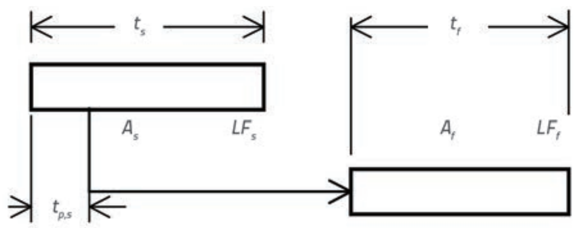
Between the activities in the PMD diagram used in this paper there are n_r links, and each link v_j connects the initial activity A_s with the following activity A_f, and so we have

- for the "start – end" link



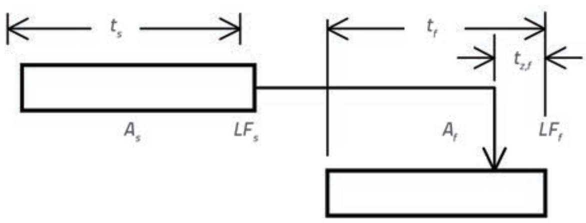
$$LF_f - LF_s \geq t_s \tag{6}$$

- for the "start – start" link, with the time interruption of $t_{p,s} \geq 0$;



$$LF_f - LF_s - t_s + t_{p,s} \geq t_f \tag{7}$$

- for the "end – end" link, with the time reserve of $t_{z,f} > 0$



$$LF_f - LF_s \geq t_{z,f} \tag{8}$$

The following is valid for the first activity A_1

$$LF_1 = t_1 \tag{9}$$

while the following applies to the last activity

$$LF_{na} = t_{pr} \tag{10}$$

Inequations (2) to (8) and equations (9) and (10) define constraints with unknown variables for duration of activities t_i and the latest completion of such activities LF_i (i = 1,2,...,n_a), and so the number of variables is n_v=2n_a. Two constraints (2) exist for each activity A_i, and for each link v_j between activities there is n_r of constraints. When we add to this two constraints (3), and one constrain (5), (9) and (10), we get the total number of constraints n_u in the PMD diagram.

$$n_u = 2n_a + n_r + 5 \tag{11}$$

2.2. Project realization costs and objective function

Project realization costs can be divided into direct and indirect costs. Direct costs are related to each activity separately, while indirect costs are related to the entire project realization phase. Direct costs cover the cost of labour, materials, energy, mechanical plant, and other resources. They are calculated separately for each activity on the project. Indirect costs contain overhead expenses, operating and contracting costs, technical and administrative personnel costs, site management costs, delay costs to be borne by the company according to the contract due to an unjustifiable delay of works, safety at work costs, etc. Indirect costs can be reduced with performance bonuses in case the contractor completes the work ahead of time. The sum of direct and indirect costs incurred during the project realization phase constitutes the total costs.

Direct costs CD_i of activity A_i reduce with the extension of the time for realization of such activity. The link between the duration of activity and direct costs can be linear, bilinear or can assume the form of a nonlinear function, as shown in Figure 1. The bilinear and nonlinear links correspond to a greater extent to a real situation, especially for longer-lasting activities. In case of linear and bilinear links between activity duration and its costs, the project duration is optimised by means of linear programming.

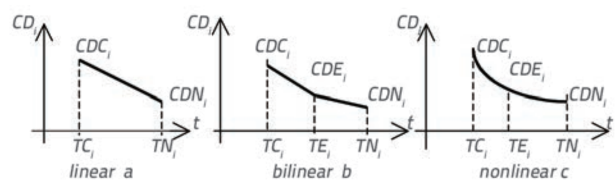


Figure 1. Link between direct costs and duration of activities

In case of linear approximation of costs according to Figure 1a, we have

$$CD_i(t_i) = CDN_i + (TN_i - t_i)\Delta CD_i / \Delta T_i$$
$$\Delta CD_i = CDC_i - CDN_i,\ \Delta T_i = TN_i - TC_i \tag{12}$$

If the time-cost approximation is made via quadratic parabola using the Lagrange interpolation formula, then directs costs of the activity A_i are:

$$CD(t_i) = CDC_i \cdot LC(t_i) + CDE_i \cdot LE(t_i) + CDN_i \cdot LN(t_i) \tag{13}$$

where

$$LC(t_i) = \frac{(t_i - TE_i)\cdot(t_i - TN_i)}{(TC_i - TE_i)\cdot(TC_i - TN_i)}$$
$$LE(t_i) = \frac{(t_i - TC_i)\cdot(t_i - TN_i)}{(TE_i - TC_i)\cdot(TE_i - TN_i)} \quad i = 1,2,\ldots,n_a \tag{14}$$
$$LN(t_i) = \frac{(t_i - TC_i)\cdot(t_i - TE_i)}{(TN_i - TC_i)\cdot(TN_i - TE_i)}$$

Indirect costs are related to the total duration of the project realization phase t_{pr}. The total time – indirect costs relationship, shown in Figure 2, is nonlinear and the linearization can be conducted, as in the preceding case, in the following way:

$$CI\left(t_{pr}\right)=CIC_i+\frac{CIN-CIC}{TN-TC}\left(t_{pr}-TC\right)$$

where *TN* and *TC* are the project realization phase completion times with normal and crashed duration of activities, while *CIN* and *CIC* are indirect costs in the project realization phase, which correspond to the times *TN* and *TC*.

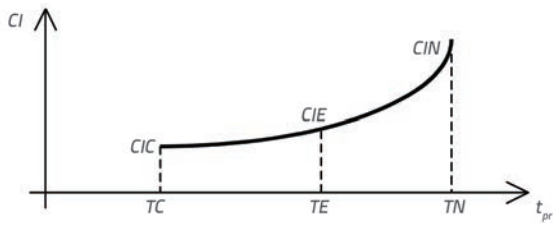


Figure 2. Link between duration of project realization phase and indirect costs

During the time t_{pr}, indirect costs can be expressed, just like the direct costs, in form of a quadratic function, as related to the costs *CIC*, *CIE* and *CIN*, as shown in Figure 2.

$$CI\left(t_{pr}\right)=CIC\cdot LC\left(t_{pr}\right)+CIE\cdot LE\left(t_{pr}\right)+CDN\cdot LN\left(t_{pr}\right) \qquad (15)$$

Here the Lagrange quotients *LC, LE* and *LN* are calculated according to expressions (14) but, instead of times TC_i, TE_i i TN_i, and the time t_i, which are related to individual activities, the calculation is made with the times *TC*, *TE*, *TN* i t_{pr} that are related to the project realization phase. The total costs for completion of the project realization phase within the time t_{pr} are:

$$CU\left(\mathbf{t},t_{pr}\right)=CD\left(\mathbf{t}\right)+CI\left(t_{pr}\right) \qquad (16)$$

where $CD(\mathbf{t})$ are direct costs, while $CI(t_{pr})$ are indirect costs. The total direct costs in the project realization phase, containing the activities n_a, are:

$$CD\left(\mathbf{t}\right)=\sum_{i=1}^{n_a}CD_i\left(t_i\right) \qquad (17)$$

where the vector is $\boldsymbol{t} = [t_1, t_2, ..., t_{na}]$. In order to determine an optimum duration of activities and the project realization phase, it is indispensable to calculate the smallest value of the total costs, expressed through the objective function (16), with the fulfilment of constraints from (2) to (10)., i.e.

$$z=\min CU\left(\mathbf{t},t_{pr}\right) \qquad (18)$$

The objective function (18) and constraints (2) to (10) define the mathematical time-cost optimization model for the project realization phase.

In the real critical path diagram, the number of constraints is often very high. In case of the linear objective function, the problem can be solved using the Simplex method for linear programming, while in the case of quadratic functions, such as the objective function (18), the problem can be solved using the quadratic programming methods. However, in addition to real variables t_i, these algorithms also require introduction of additional variables, and so their number in the simplex-matrix becomes even greater, which is why these method can not be recommended. This is the reason why many authors apply acceptable computation methods that are mentioned in the first section of the paper. The optimization based on the particle swarm method, applied in this paper, is presented in the following section.

3. Determination of optimum time and costs using particle swarm optimization

The particle swarm optimization method is an evolutive computation method based on populations [39]. It can be used for optimization of continuous objective functions with and without constraints. Together with the ant colony optimization method, bee colony optimization method, and stochastic diffusion search method, it belongs to the group of swarm intelligence methods. These methods are based on socio-psychological principles. Swarm intelligence systems are made of populations composed of simple members (particles) which interact with one another, and also with their surroundings. They observe and understand their surroundings and can take actions that maximise the possibility of success. These systems include bee swarms, bird flocks, fish schools, animal herds, bacteria groups, etc. The method is inspired by the similarity of their behaviour to the socio-psychological behaviour of human beings when solving various problems. When decisions have to be made to solve a practical problem, the decision maker discusses the issue with other people, gathers information, advice, and opinion of others, and applies his/her knowledge and experience. The decision maker particularly takes into account his most successful solution to this or similar problem in the past, and the most successful solution obtained by other people from his close or wider surroundings, with whom he has exchanged opinions and whose experience he has applied. When looking for food or when migrating to other destinations, bird flocks, fish schools, or bee swarms adjust their physical movements and exhibit a sort of social behaviour. To overcome various obstacles and avoid dangerous situations, they use their senses to exchange information, and every member of the swarm acts in accordance with its position with regard to other members, using its previous "experience" and the most favourable instant position of a swarm member, or a member that leads the swarm. Unlike bee swarms or fish schools, groups of people use their cognitive and creative capabilities in their interactions with others.

The particle swarm optimization method "involves a very simple concept, and paradigms may be implemented in several

computer code lines. It requires simple mathematical operators, and is inexpensive with regard to memory and speed" [39]. This method for solving nonlinear programming or optimization problems, as will be seen below, is very simple from the mathematical and algorithmic standpoints, and it provides in many instances highly accurate results. That is why it presents some advantages for solving a greater number of problems when compared to some other more complex heuristic methods, such as the genetic algorithms, evolutive strategies, algorithm of simulated annealing, etc., and also when compared to traditional numerical methods based on the theory of mathematical programming. The main advantage of this method - compared to genetic algorithm method and other evolutive methods - is a simple implementation, as it involves only several parameters that have to be adjusted in an iterative optimization process [40].
This method simulates initial solutions using the Monte Carlo simulation, and these initial solutions are improved through simulations made during subsequent iterations, until an optimum solution is found. This is why it is ranked among stochastic and heuristic methods.
In the scope of the time-cost optimization problem, it is necessary to determine an optimum duration of activity t_i and project realization phase t_{pr}, so as to obtain minimum values of the objective function z, which represents the total costs in the project realization phase, and which can be presented, due to (18), as follows:

$$z = \min CU(\mathbf{t}) = \min f(\mathbf{t}), \quad \mathbf{t} = [t_1, t_2, ...t_{na}], \quad \mathbf{t} \in R^{na} \tag{19}$$

provided that constraints from (2) to (10) are fulfilled. These constraints determine the set of allowable solutions, *D.* The optimization procedure is conducted iteratively.

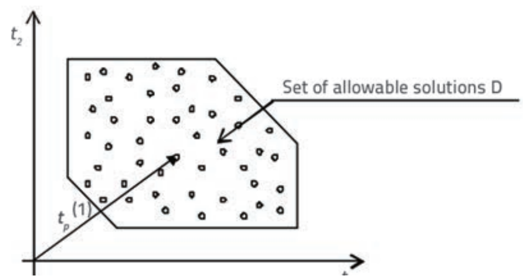


Figure 3. Simulated initial solutions

In the first iteration, which is called initialisation, the Monte Carlo method is used to simulate, in the set of allowable solutions *D*, the vectors $\boldsymbol{t}_p^{(1)}$ that represent the rows of the matrix $\mathbf{T}^{(1)} = [t_{p,i}^{(1)}]$, $i = 1, 2, ..., n_p$ in the Euclidean space R^{na} (cf. Figure 3). They represent initial position vectors of the number n_p of swarm members located in the set *D* in which they move, and whose components are compliant with the constraint (2).

$$t_{i,p}^{(1)} = TC_i + (TN_i - TC_i) \cdot (rnd) \quad i = 1,2.,,,,n_a; \; p = 1,2,....n_p, \; 0 < (rnd) < 1 \tag{20}$$

where (*rnd*) is the simulated random number of uniform distribution, n_a is the number of activities, while n_p is the number of swarm members.

These expressions confirm fulfilment of the constraint (2) in the first iteration. The earliest $EF_{i,p}^{(k)}$ and the latest $LF_{i,p}^{(k)}$ completion of activities A_i (i = 1,2,..,n_a), and duration of the project realization phase $t_{pr}^{(k)}$, will be determined for the simulated time $t_{pr,p}^{(k)}$ (p = 1, 2, ..., n_p) in this and in the subsequent iterations k using the critical path method (CPM). This fulfils the remaining constraints, as the calculation of duration is based on formulas used in the critical path method (6), (7), and (8). Direct costs, indirect costs, and total costs, representing the objective functions $f_p^{(1)} = f(\boldsymbol{t}_p^{(1)})$, are determined according to formulas (13) to (17) for the obtained durations of the project realization phase $t_{pr,p}^{(k)}$ (p = 1, 2, ..., n_p). Simulated vectors $\boldsymbol{t}_p^{(1)}$ are analysed to select the one for which the objective function $f_p^{(1)}$ has the lowest value, and this vector represents the best global solution $\mathbf{t}_g^{(1)}$ in the first iteration, where the following is valid

$$\min f_p^{(1)} = z^{(1)} = f\left(\mathbf{t}_g^{(1)}\right) \tag{21}$$

In a subsequent iteration k = 2, 3, 4, ..., and for the know position vector $\boldsymbol{t}_p^{(k-1)}$, the vector showing position of the swarm member $\boldsymbol{t}_p^{(k)}$ is determined (as shown in Figure 4) according to the following expression

$$\mathbf{t}_p^{(k)} = \mathbf{t}_p^{(k-1)} + \mathbf{v}_p^{(k-1)}; \; k = 2,3,...; \; p = 1,2,....n_p \tag{22}$$

where $\boldsymbol{v}_p^{(k-1)}$ is the change of the vector showing position of the member of the swarm p after iteration k-1. This vector is also called the speed vector and is calculated according to the following formula

$$\mathbf{v}_p^{(k-1)} = \omega \mathbf{v}_p^{(k-2)} + \phi_1 \left[\mathbf{t}_{l,p}^{(k-1)} - \mathbf{t}_p^{(k-1)}\right] \cdot (rnd) + \phi_2 \left[\mathbf{t}_g^{(k-1)} - \mathbf{t}_p^{(k-1)}\right] \cdot (rnd); \quad 0 < rnd < 1 \tag{23}$$

where $\boldsymbol{t}_{l,p}^{(k-1)}$ is the best position of the member (particle) of the swarm p that can be related to the smallest objective function in the previous iterations 1,2,...k-1. This vector or component is called the *data component* and it introduces in the calculation the information about the best position each swarm member assumed in the past. $\boldsymbol{t}_g^{(k-1)}$ is the position vector of that member (particle) of the swarm p for which the objective function has the smallest value $f(\boldsymbol{t}_p^{(k-1)})$ in the iteration k-1. This vector is also called the *social vector* or *social* component, and it takes into consideration the best instant position of the said swarm member.
The factor ω is the *inertia factor* [41] and its value is $\omega \leq 1$. It is often assumed to be ω = 0.9 and it influences reduction of the speed change vector during subsequent iterations. The computation with values 0.7 or 0.8 is recommended. It is also recommended to take at the beginning of the iterative process a higher inertia factor value so as to increase efficiency of the search for the global solution [33].
Factors φ_1 and φ_2 are called *learning factors* and they determine the relative influence of the data component about the best position of each particle p in the swarm $\boldsymbol{t}_{l,p}^{(k-1)}$ and the so called *social component* $\mathbf{t}_g^{(k-1)}$ on the position $\mathbf{t}_p^{(k)}$ of the member of the

swarm p during the subsequent iteration k. These coefficients are $\varphi_1 \approx 2$, $\varphi_2 \approx 2$.
Value (*rnd*) is the random number of uniform distribution, and every time a different value is simulated in the above expression. In order to prevent an excessively fast movement of some particles in the zone of allowable solutions, which could lead to divergence in the optimization process, it is recommended to limit the speed [42], so that the maximum absolute value of speed component in the iteration k is smaller than or equal to a specified maximum speed v_{max}, i.e.:

$$\max\left|v_{j,p}^{(k)}\right| \le v_{\max}; \quad p = 1,2,....n_p; \quad k = 2,3,.... \tag{24}$$

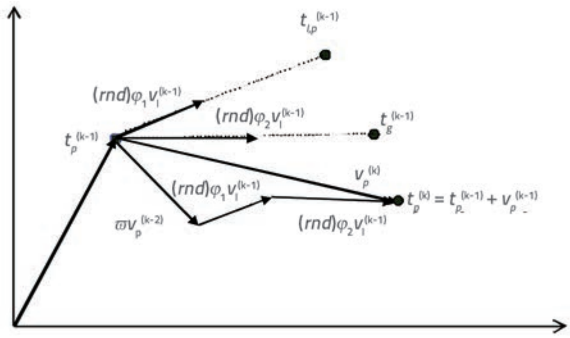


Figure 4. Definition of new position vector $\mathbf{t}_p(k)$ for a particle in the swarm p

In literature, some authors recommend that this value should amount to 10 percent of the maximum value d_j

$$d_j = \max t_j - \min t_j; \quad j = 1,2,....n \tag{25}$$

The constriction factor K is introduced in order to limit the speed and prevent divergency or the so called process "explosion". This factor is calculated according to the following expression [43]

$$K = \frac{2}{\left|2 - \phi - \sqrt{\phi^2 - 4\phi}\right|}, \qquad \phi = \phi_1 + \phi_2 \ge 4 \tag{26}$$

This factor influences reduction of the change of position, or speed, of swarm particles in the following iterations, and so it is calculated according to the following expression:

$$\mathbf{v}_p^{(k-1)} = K\left\{\mathbf{v}_p^{(k-2)} + \phi_1\left[\mathbf{t}_{l,p}^{(k-1)} - \mathbf{t}_p^{(k-1)}\right]\cdot(rnd) + \phi_2\left[\mathbf{t}_g^{(k-1)} - \mathbf{t}_p^{(k-1)}\right]\cdot(rnd)\right\} \tag{27}$$

The vector $\boldsymbol{t}_{l,p}^{(k)}$ that corresponds to the "best" minimum objective function value for a particle in the swarm p in all preceding iterations, including the iteration k, is calculated by comparing objective functions for that member in two neigbouring iterations $k-1$ and k, in the following way:

$$\mathbf{t}_{l,p}^{(k)} = \mathbf{t}_{l,p}^{(k-1)} \text{ za } f_p^{(k)} > f_p^{(k-1)}, \ \mathbf{t}_{l,p}^{(k)} = \mathbf{t}_{l,p}^{(k)} \text{ za } f_p^{(k)} < f_p^{(k-1)} \tag{28}$$

$p = 1, 2, ..., n_p$, $k = 2, 3, 4, ...$

In the first iteration $k = 1$, we have $\boldsymbol{t}_{l,p}^{(k)} = \boldsymbol{t}_g^{(1)}$ i $\boldsymbol{v}_p^{(1)} = 0$, $p = 1,2,...,n_p$.

According to expressions (26) and (27), parameters φ_1 i φ_2 influence the speed vectors $\mathbf{v}_p^{(k)}$, i.e. the change of position of a member (particle) in the swarm p or, in other words, its trajectory of movement in the Euclidean space R^{na}. If these parameters are close to zero, the movement trajectory tends to exhibit smooth curved lines [39], because the changes in iterations are small. After several iterations, trajectories move toward best positions or solutions. The inertia factor and maximum values of parameters φ_1 i φ_2 are not independent [43], and the use of their pairs is recommended: $\omega = 0.7$, max φ_1=max φ_2 = 1.47 or ω = 0.8, max φ_1 = max φ_2 = 1.62.
Once the components $t_{i,p}^{(k)}$ (i = 1, 2, ..., n_a) of the vector $\mathbf{t}_p$ are determined, it is necessary to check whether they meet constraints(2). If $t_{i,p}^{(k)} < TC_i$ then we have $t_{i,p}^{(k)} = TC_i$, and if $t_{i,p}^{(k)} > TN_i$ then we have $t_{i,p}^{(k)} = TN_i$. These durations of activities in iteration k are used to calculate, using the critical path method, the earliest and the latest completion of activities and duration of the project realization phase $t_{pr,p}^{(k)}$, as well as the direct and indirect costs, and the objective function for each swarm member and for each iteration k = 1,2,3,...

$$f_p^{(k)} = f\left(\mathbf{t}_p^{(k)}\right), \ p = 1,2,....n_p \tag{29}$$

An important constraint is not met if the calculated project realization time $t_{pr,p}^{(k)}$ for the particle p in the iteration k is greater than project realization time specified in the contract t_{ug}, i.e. $t_{pr,p}^{(k)} > t_{ug}$ (5). In this case, it should be specified for this particle p, i.e. for such project realization time, that the total costs have a very high value. In this way, this particle p is excluded from further procedure, as it can not fulfil the next condition for selection of the vector $t_g^{(k)}$. After that, the minimum of these values is defined in each iteration

$$z^{(k)} = \min f\left(\mathbf{t}_p^{(k)}\right) = f\left(\mathbf{t}_g^{(k)}\right), \ k = 1,2,.... \tag{30}$$

as well as the vector $t_{pr,p}^{(k)}$ that corresponds to this iteration.
The procedure is repeated until the absolute value of the difference between the minimum objective function values in two subsequent iterations becomes negligible.

$$\left|z^{(k)} - z^{(k-1)}\right| \le \delta \tag{31}$$

where δ is a small number selected in advance, depending on the desired solution accuracy.

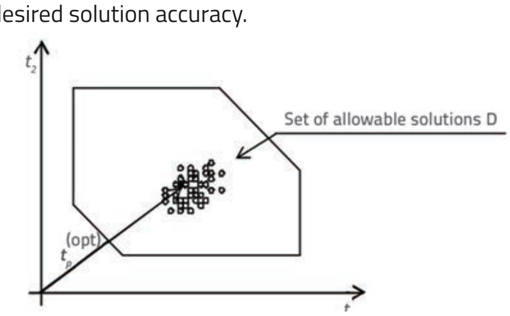


Figure 5. Optimum solution

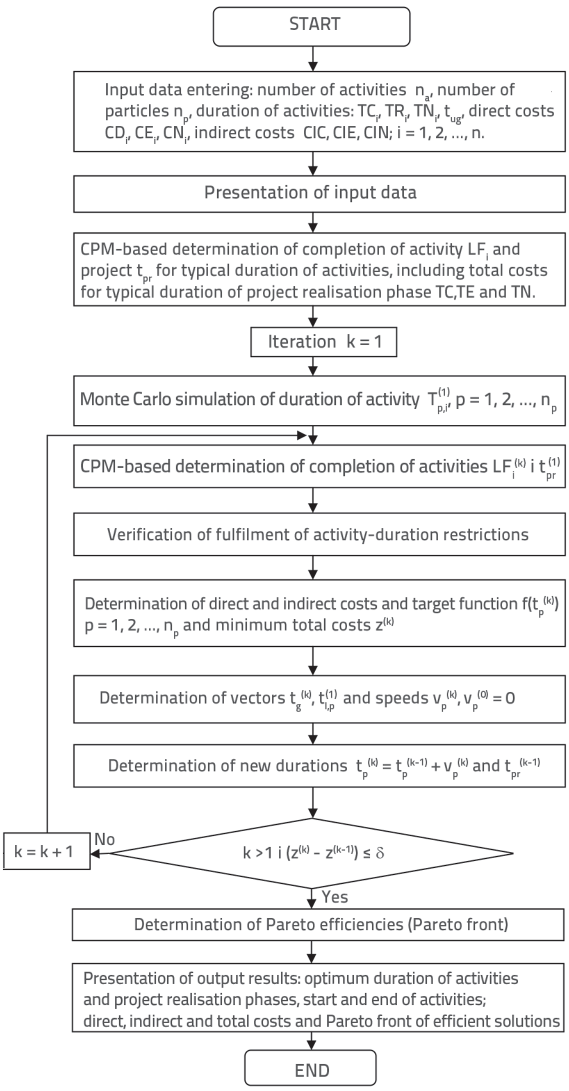


Figure 6. Process algorithm based on particle swarm optimization

Thus obtained solution for the duration of realization of activity $t^{op} = [t_1^{op}, t_2^{op}, ..., t_{na}^{op}]$ and for the project realization phases t_{pr}^{op}, with the corresponding direct costs $CD(t_{pr}^{op})$, indirect costs $CI(t_{pr}^{op})$, and total costs $CU(t_{pr}^{op})$, is accepted as an optimum solution. The position of swarm particles corresponding to an optimum solution is presented in Figure 5. The algorithm of the presented procedure is shown in Figure 6.

Based on this procedure, the authors of this paper developed an appropriate computer program called OPTCOST_PSW using the program system MATLAB.

4. Pareto front: time – costs

If in the interval of the shortest and normal project realization time [*TC*,*TN*] the value t_{ug} is changed for the constraint (5) in such a way that it represents the shortest (minimum) time within which the project realization phase must be completed, and if the minimum total costs $CU(t_{ug})$ for the project realization phase are calculated for each such value using this optimization method, then we obtain the dependence function CU_{min} of $t_{ug} = t_{min}$, which is shown in form of curve in Figure 7. Points with the coordinates (t_{min}, CU_{min}) are shown in this curve.

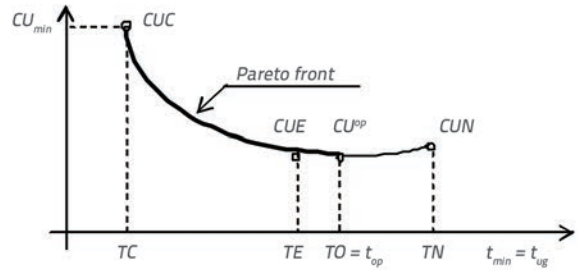


Figure 7. Pareto front of non-dominant (efficient) solutions

If from the standpoint of multicriteria optimization the total costs CU_{min} are taken to be one criterion, and if the shortest project realization time t_{min} is taken to be the other criterion, then this minimum time – minimum costs curve constitutes the Pareto front for the interval [*TC*, *TO*], and this front is formed of non-dominant (efficient) two-criteria optimization solutions. These solutions are characterized by the fact that total costs CU_{min} reduce with an increase in minimum time t_{min} and, vice versa, if the time t_{min} is reduced, the total minimum costs CU_{min} are increased. The curve points above the time interval [*TO*,*TN*] are not Pareto front points as the total costs CU_{min} increase with an increase in time t_{min}. This curve is important for decision makers, i.e. for the clients and contractors alike, as it can be used to monitor the change of costs over time during the project negotiation phase.

5. Implementation measures

An appropriate team must be formed for planning and checking realization of construction projects. This team must be formed of specialists who are familiar with planning techniques, but also with construction technologies and financial and organisational problems related to the realization of works. The team should make a realistic plan based on the critical path diagram showing realization of works, taking into account specified types of works, conditions under which such works will be realized, necessary and available human, financial and material resources, and time available for the realization of the project. Typical duration of activities and project realization time, including relevant direct and indirect costs, must be determined using an appropriate IT system, production standards related to the use of labour, material, mechanical plants and money, and experience in the realization of other projects. In addition to these input data, the information about correlation of activities should also be entered into the computer program. The processing of input data based on this method results in an optimum duration of activities and in an optimum project realization phase, with the corresponding minimum costs. These output results should however be analysed to assess their accuracy and applicability. In case some false or unrealistic results are noticed, the input

data must be verified, appropriate changes to such input results should be made, and new solutions should be obtained, which must once again be verified. It is recommended to vary input data for some activities depending on their costs and durations in some intervals, and to accept those times which have been found to be most suitable during processing or results. The total project realization costs do not greatly deviate from the minimum (optimum) costs in a wider time interval, which is very favourable for the project realization planning, as all times within this interval can be accepted as optimum times. The use of these optimization methods is recommended for the global time scheduling with a smaller number (up to one hundred) larger activities, based on which detailed plans can be elaborated.
The project activities and project duration information obtained in this way can also be used as input data for complex program packages, such as the MS PROJECT, PRIMAVERA, etc.

6. Example

The time schedule with 14 activities relating to the construction of a small structure (filling station) is shown in Figure 8 [44]. The optimization was made using the particle swarm method presented in this paper. The list of activities is given in Table 1, and typical duration of activities and relevant costs are presented in Table 2.

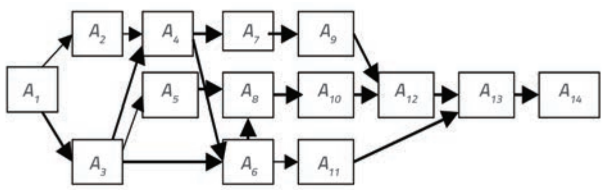


Figure 8. Critical path diagram

Table 1. List of activities

Activity A_i	Designation of activity
1	Site preparation activities
2	Supply of materials
3	Earthworks
4	Rough construction works
5	Supply of equipment (tanks and filling pumps)
6	Road bed construction for access road
7	Prefabricated works
8	Excavation work for tanks
9	Tank covering works
10	Masonry works for tanks
11	Asphalt pavement for access road
12	Assembly of pumps and tanks
13	Finishing works
14	Final inspection and initial operation

Table 2. Typical time – cost predictions for activities

A_i	TC_i [days]	TE_i [days]	TN_i [days]	CDC_i [EUR]	CDE_i [EUR]	CDN_i [EUR]
1	4	6	9	18,000	12,500	8,000
2	2	3	5	10,000	6,400	4,500
3	2	4	6	12,600	8,600	5,000
4	7	10	15	30,000	20,000	14,000
5	4	6	9	18,000	12,000	8,000
6	1	2	3	6,200	4,000	2,000
7	2	3	5	10,800	6,800	4,000
8	2	4	7	14,000	8,000	4,000
9	4	6	9	18,200	13,000	8,700
10	2	3	5	11,000	6,400	4,000
11	3	5	7	14,300	10,000	6,100
12	2	4	7	8,400	8,400	4,800
13	1	2	4	2,000	2,000	1,500
14	1	2	3	1,000	1,000	500

Based on CPM the following information was obtained for the above input data: shortest time TC = 23 days, conventional time TE = 37 days, and normal time TN = 59 days, for the completion of the project. Indirect costs for these construction times amount to: CIC = 15.000 €, CIE = 24.500 € and CIN=56.500€.
The completion time specified in the contract is t_{ug} = 55 days.
The following results were obtained using the program COSTOPT_PSW:

- optimum duration of activities in days
  $\mathbf{t}^{op}$ = [8, 5, 6, 12, 9, 3, 5, 7,9, 4, 7, 5, 2, 1]

- optimum earliest activity completion times
  EF^{op} = [8,13, 14, 26, 23, 29, 31, 36, 40, 40, 36, 45, 47, 48]

- optimum latest activity completion times
  LF^{op} = [8, 14, 14, 26, 29 ,29, 31, 36, 40, 40, 45, 45, 47, 48]

- optimum time for realization of project t_{pr}^{op} = 48 days
- all activities are on critical paths, except for the activities A_2, A_5 and A_{11}
- number of swarm particles n_p = 50, number of iterations n_{it} = 14
- optimum project realization costs are:
  - direct costs CD^{op} = 81.847 EUR,
  - indirect costs CI^{op} = 37.892 EUR,
  - total costs CU^{op} =119.739 EUR.

The same results were also obtained using the genetic algorithm method.
Total minimum construction costs, representing Pareto front points of efficient solutions are presented in Table 3 for some typical values of minimum (contract-specified) construction times (Figure 7).

Table 3. Pareto front points

t_{min} [days]	CU_{min} [EUR]
23	184.000
30	150.910
35	136.009
37	129.313
45	120.769
48	**119.739**
59	131.600

The last point (59;131.600) is not optimal in relation to Pareto front.

The authors tested the optimization method and computer program presented in this paper on two additional time-cost optimization examples. In these examples, the optimization was made using the Fondahl heuristic method [38], i.e. the genetic algorithm method [25]. In both tested cases, the results obtained using the particle swarm method correspond well with the results obtained in [25, 38] for similar time and cost input data.

7. Conclusion

The particle swarm optimization method can successfully be used for optimising realization of construction projects. The authors have adopted this method for solving these problems, taking into account project costs, activity durations, and activity correlations in the critical path diagram. They also developed an appropriate computer program. Nonlinear dependence between the costs and project realization time and project activities, as used in this paper, provides results that are much more realistic when compared to linear dependence, which was used in initial phases of development and application of the time and cost project optimization methods. The proposed procedure, considering its simplicity and level of accuracy, provides good results and presents several advantages compared to methods based on the use of simplex algorithms for linear programming, and other traditional mathematical programming methods. This method also presents some advantages over evolutive methods: genetic methods, algorithms, evolutive strategies and other mathematical programming methods, as it enables easier elaboration of computer programs while providing sufficiently accurate results. The use of this method is recommended for optimization of global critical path diagrams for projects with a smaller number of project-significant activities. Using their computer programs, the authors compared, using literature examples and their own examples, the results obtained by the method presented in this paper, with genetic algorithms and heuristic methods presented by Fondahl and Trbojević, and have obtained similar highly accurate results.

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