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Bridge condition forecasting for maintenance optimisation

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Preliminary note

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Bridge condition forecasting for maintenance optimisation

The bridge management system used on the national road network of the Republic of Croatia does not contain the deterioration forecasting model that could be used for planning future remedial activities. A database on the condition of bridge elements has nevertheless been compiled, containing numerical values of the degree of deterioration. Results obtained by analysing usability of three deterioration models are presented in the paper in order to estimate which one would be the most favourable for Croatian conditions. The model based on the Markov process was found to be the most appropriate.

Key words:
Bridge management system, Markov process, bridge assessment, bridge maintenance, bridge deterioration

Prethodno priopćenje

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Predviđanje stanja mostova radi optimalizacije održavanja

Sustav gospodarenja mostovima na mreži državnih cesta Republike Hrvatske ne sadrži model za prognoziranje dotrajavanja koji bi se upotrijebio pri planiranju budućih popravaka, ali je prikupljena baza podataka o stanju, odnosno numerički izraženom stupnju oštećenosti elemenata mostova. Rad prikazuje rezultate istraživanja uporabljivosti triju modela dotrajavanja u nastojanju da se ocijeni koji je od njih za naše okolnosti najpovoljniji. Najprikladnijim je ocijenjen model koji se zasniva na Markovljevom procesu.

Ključne riječi:
sustav gospodarenja mostovima, Markovljev proces, ocjenjivanje mostova, održavanje mostova, dotrajavanje mostova

Vorherige Mitteilung

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Zustandsvorhersage von Brücken zur Erhaltungsoptimierung

Das Verwaltungssystem für Brücken des nationalen Straßennetzes in Kroatien umfasst derzeit kein Model für die Schadensprognose, das in der Planung zukünftiger Wartungsarbeiten angewandt werden könnte. Dennoch ist eine Datenbank zusammengestellt worden, die numerische Angaben der Beschädigungsgrade beinhaltet. In der vorliegenden Arbeit ist die Eignung drei verschiedener Beschädigungsmodelle für die gegebenen Bedingungen in Kroatien untersucht worden. Das auf dem Markov-Prozess beruhende Model ist als angemessenstes beurteilt worden.

Schlüsselwörter:
Verwaltungssystem für Brücken, Markov-Prozess, Zustandsbewertung von Brücken, Brückenwartung, Beschädigung von Brücken

1. Introduction

Management of structures includes activities which tend to optimise the use of an infrastructure facility, so that the benefits are maximised over a predetermined period of time. The bridge management has been developing by Croatian national road operator – Croatian Roads Company (Hrvatske ceste d.o.o.) since 1995, when the bridge management system (BMS) called HRMOS, based on the system used by the Danish Road Administration [1], was introduced. Over the past period of little less than two decades, data have been gathered about condition of bridge elements, and these data now enable preparation and calibration of bridge-condition forecasting models. The main problem in model development is estimation of the rate at which the damage causing process is advancing, based on a modest number of consecutive condition state measurements, or damage level measurements. These condition states are described through qualitative properties: type, cause of occurrence, and predictable damage advancement rate, and also through quantitative data such as the area of the bridge element that is affected by damage. The study of efficiency of the regression model, model using Markov chain, and model based on homogenous Markov process, is presented.

Croatian Roads Company (HC) operates the national road network 6585 km in length, with more than 1538 bridges of more than 2 metres in span. According to the Croatian Public Roads Act [3], HC is required to keep a uniform road data base so as to ensure proper technical & technological uniformity of the public road network. This database includes all data on bridges situated along public roads (including motorways, county and local roads), which has encouraged us to develop a uniform model enabling long term planning of maintenance activities on the strategic level. As such model includes bridges that fall under authority of several road operators, it has to be simple and transparent. In fact, various bridge management systems use different methods to reach their objectives, depending on local conditions and significance of facilities, level of technical education, property management tradition, property-right relationships, and organisation of competent services. The main limitation hampering system development is the difficulty to obtain efficient indicators for defining remedial work priorities and financial planning, based on a limited database.

Deterioration models can be prepared based on Principal Bridge Inspections which have to be carried out, according to prevailing regulations, at least once every six years. At that, the visual inspection technique is used, and the results are presented as numerical ratings of 13 bridge elements, and the raring of the entire bridge. Bridge ratings range from 1 to 5. Rating 1 means that there is no damage or that the damage is negligible, while the highest rating 5 means that the damage is such that the

Table 1. Bridge data prepared for analysis, as an extract from the BMS kept by Croatian Roads Company (HC) – an example of basic data and inspection result data

General bridge data							Numerical assessment of bridge condition, from 1 to 5					
Bridge name	Year of construction	Total length (m)	Total width (m)	Number of spans	Maximum span (m)	Year of inspection	Area	Abutments	Piers	Bearings	Deck slab	Bridge, general
						2002.	1	1		1	3	3
						2009.	1	1		1	4	4
Sigetac Ludbreški	1988.	44,0	9,9	3	16,0	1998.	1	1	1	1	1	2
						2000.	2	1	1	1	1	2
						2003.	2	1	1	1	1	2
						2008.	3	2	2	2	2	2
NV Frigis	1979.	64,0	12,6	4	15,3	1997.	1	2	2	1	1	2
						2002.	1	2	2	2	2	2
						2008.	2	2	2	2	2	2
NV Klajnova	1971.	61,0	12,6	4	15,3	1996.	2	2	2	1	3	3
						2002.	3	2	2	2	4	3
						2008.	2	2	2	2	4	3
Drnje	1930.	36,0	5,6	1	11,5	1998.	1	1		1	1	2
						2003.	3	2		1	3	4
						2009.	3	2		1	3	4

element is practically unusable. The inspection methodology and its limitations are described in paper [2] where a continuous improvement of the visual inspection system is proposed.

The structure of bridge data used in this paper is presented, as an extract from the mentioned database, in Table 1.

U radu su analizirane ocjene stanja rasponskog sklopa (u radu se rabi termin "ploča" preuzet iz sustava gospodarenja Hrvatskih cesta) jer jedinstvena ocjena cijelog mosta ne pruža cjelovit podatak. Naime, prema protokolu kojim se koristi HRMOS pravilo je da ocjena čitavog mosta ne može biti viša od one dodijeljene najoštećenijem dijelu (skupini dijelova) mosta niti niža od one dodijeljene bilo kojoj skupini dijelova mosta, no u cjelini je njezino dodjeljivanje vrlo subjektivno.

In this paper, the focus is on the condition of superstructure (the term "deck slab" taken from the HC management system is used) because a single rating for the entire bridge can not be taken as a consistent information. In fact, according to the HRMOS protocol, the rating for the entire bridge can not be higher that the one assigned to the most damaged component (group of parts), nor lower than the one assigned to any group of parts of the bridge. As a whole, general rating of a bridge is highly subjective.

2. Forecasting future condition state of bridges

2.1. General

The deterioration forecasting problem can be formulated as follows: it is necessary to develop a theoretical model that will describe the degradation process and enable prediction of bridge deterioration over time, taking at that into account the natural environment, and the way in which the facility has been used and maintained.

Figure 1. Bridge managed by Hrvatske ceste prior to repair in 2012 – typical damage to cornice and superstructure

The road network contains many bridges of various age and condition. The majority of these bridges are small-size concrete bridges [2]. Four Principal Inspections, spaced at approximately 4 year intervals, have so far been conducted in the scope of systematic management based on a uniform methodology. A statistical sample of 107 bridges inspected since 1996, on which no significant repairs have been made, has been singled out (Figure 1).

In addition to the use of inspection data, the following initial assumptions were also adopted for model development:

- bridge element ratings obtained through regular inspections roughly coincide to condition states that can be related to specific repair procedures, or typical cost estimates,
- principal degradation processes on structures of similar type are the same, and the deterioration progresses at a similar rate, which is why it is reasonable to look for a statistical relationship between the bridge age and condition,
- a one-directional deterioration process, advancing from lower (better) to higher condition states, is considered,
- bridges on which no significant structural repairs had been made were considered in the analysis (the influence of repairs was not modelled).

2.2. Development of prognostic models

Deterioration models can basically be divided into mathematical (statistical), empirical and physical models. Statistical models are formed by analysing data that describe condition of a greater number of bridges, empirical models are based on experience, while physical models are based on knowledge and modelling of damage-causing processes. Numerous BMS use various deterioration models [5] but their common feature is that they all have a calibration capability, i.e. the possibility of subsequent adjustment of model based on information gained during the deterioration monitoring process.

Regression mathematical models have been developing since 1980s, i.e. since the time when the databases on completed inspections were already accumulated. Linear and nonlinear models for forecasting future processes on groups of bridges characterized by common features have been eveloped [6]. The US studies of dependence of bridge condition state on bridge age have resulted in typical curves (Figure 2) in which typical phases in the life span of bridges can be recognised:

- steep fall during the first 20 to 25 years,
- almost horizontal or slightly ascending trend between the years 25 and 45,
- mild fall between the years 45 and 60.

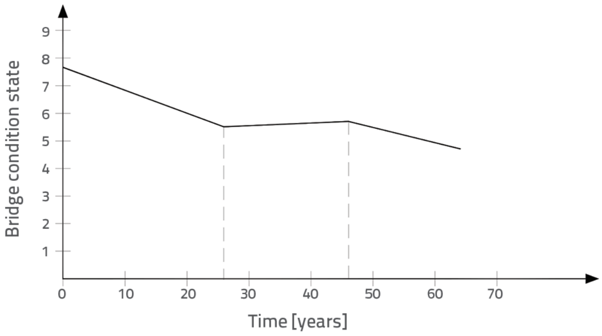

Figure 2. Dependence of bridge condition state on bridge age, according to data from the US National Bridge Register (ratings range from 9, which means no damage, to 1 which is the worst condition) [7]

This distribution of ratings by years can be interpreted as follows:

- Hidden defects, attributed to errors in bridge design and construction, are activated in the first 25 years.
- In the period from year 25 to year 30 first major repairs are made, which slightly increases an average rating, and it becomes more stable. Possible long term processes cause latent damage, which is in most cases not identified during inspection.
- As to bridges older than 45 years, it can be observed that they were generally designed according to lower standards (load-related regulations, regulations on the design of structural elements), and so they are affected by a greater level of damage.
- Bridges older than 60 years can not be regarded as a relevant sample.

The exponential function for simulating the curve form is used in statistical regression models, and some of these models also feature a sudden rise – improvement due to repair around the year 30 of service life.

The bridge management system used in Japan [8] contains time-related deterioration curves which define the remaining service life of individual bridge elements. The curves are based on theoretical formulas by which various deterioration mechanisms are modelled. More specifically, the degradation of concrete elements due to carbonation and chloride penetration is modelled by formulas that have been adopted by the Japanese society of civil engineers. Theoretical calculation results are compared with measurement results, and deterioration curves are calibrated. Formulas by which deterioration due to fatigue of concrete slabs is estimated are also used, and the introduction of formulas for modelling deterioration due to freezing, chemical influences, and alkali-aggregate reactions, is also planned. Therefore, theoretical models relating to deterioration processes are calibrated by data gained through experience.

The bridge deterioration model used by the BMS system, as developed for Croatian highway operator – Croatian Motorways Company (Hrvatske autoceste d.o.o.), also makes use of deterioration curves, which link the age of the bridge with the extent of damage [9]. The inspection reveals the type, phase, position and size of damage, in accordance with an appropriate description from the catalogue. Limit points of each damage progress phase are defined by measurable values. Curves, initially created on the basis of experience and literature, should be calibrated in accordance with test results.

One of the best known commercial bridge management systems is Pontis [10]. It is most widely used in the US where it was developed under the auspices of the American Association of State Highway and Transportation Officials (AASHTO). The system has also been adopted in one of Croatia’s neighbouring countries - Hungary. It contains statistical deterioration models based on the Markov chain theory, which is the most commonly used stochastic model for deterioration process. The model incorporated in a computer algorithm defines the probability of transfer of an element part from a better to a worse condition state. Model parameters initially incorporated in the computer algorithm contain engineering estimates of the time an element part will remain in a specific condition, and later on the data are upgraded by statistical analysis of ratings.

The model using a homogenous Markov process with the finite set of condition states and a continuous parameter, described in [4], is used as it presents some advantages compared to Markov chain model, i.e. it enables clearer interpretation of measured or observed data. This model is not used in commercial programs for infrastructure management.

3. Analysis of inspection results for typical bridges

3.1. Sample for statistical analysis

The analysis was conducted on the sample separated from the most common group of bridges in the database. Data on condition state rating was used for superstructure (deck slab) of reinforced-concrete girder bridges ranging from 10 to 80 years in age, which are subjected to various climate conditions. The sample contains 107 bridges with one or more spans, measuring five meters or more in length. Ratings obtained by visual inspection in the period from 1996 to 2012 were analysed. Due to time gap in realization of principal inspections, the data were classified in three time periods from 1996 to 2012. Only bridges on which no significant repairs have been made during their service life were taken into consideration. Figure 3 presents a histogram of mean superstructure condition ratings, which shows a mild worsening of properties in the period form 1996 to 2012.

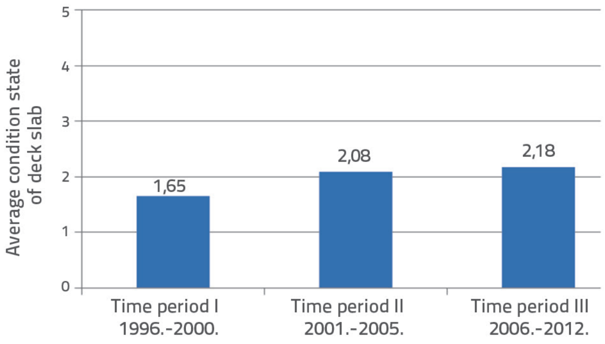

Figure 3. Mean condition ratings for superstructure (element: "deck slab") for a sample of 107 similar bridges on national roads of the Republic of Croatia

3.2. Regression model

The regression model used is a one-dimensional regression model in which the time t is a non-random variable, while the building rating S is a random variable. The form of the regression function μ is also assumed with parameters determined on the basis of statistical data, i.e. based on the set of n paired measurements $(t_1, S_1), (t_2, S_2),\dots (t_n, S_n)$ of values t i S. The regression function graph (regression curve) clearly shows the time dependence of the phenomenon under study. In a given moment, the regression function value is an expected (mean) value of the studied phenomenon, which means that in reality deviations are possible, and these deviations can also be theoretically estimated by means of regression model.

Based on models developed in the US [6], attempts were made to use the exponential regression curve, but the result has revealed that the straight line can equally be used for the given set of data (Figure 4).

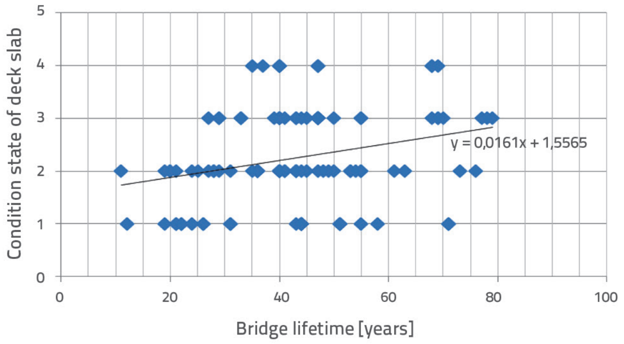

Figure 4. Regression model showing deterioration of superstructure on concrete bridges on national roads of the Republic of Croatiaa

The model shown in figure 4 can be interpreted as follows: at the tenth year of service life the bridge will be in the condition state of either 1 or 2. The condition state 2 is the most probable when the bridge is about 40 years old, after which the bridge gradually reaches the condition state 3. Most 80 year old bridges will be in condition state 3, after which there are no data that would enable further analysis.

An average rating after ten years of service life amounts to 1.6 while the rating is 2.8 after 80 years of service life, i.e. the rating changes by about one level of damage. This result – the shift for one level only in the period of seventy years - does not correspond to practical experience, which shows that bridges deteriorate at a much faster rate. This occurrence can be explained as a model weakness, but also by the fact that the road (and bridge) operator has made some repairs in the past period (regular maintenance activities) which were not entered in the database. Thus, when the regression model is used, the sample can be made only of such bridges or bridge elements that have not been subjected to significant repairs.

3.3. Markov chain model

Markov processes describe physical reality in which changes occur over time in a random manner, so that the probability of a future condition of a process is only dependent on the present condition of the process, while it is unrelated to the process development in the past [11]. In this case, the basic assumption is used when the representative sample of bridges is formed for statistical analysis. The sample may include all bridges or bridge elements on which no significant repairs have been made in the period from 1996 to 2012. In theory, it is not important for the model whether or not bridges were repaired prior to 1996.

The Markov chain is a special case of Markov process with a discrete parameter [9], and this parameter is the time in this particular case. The Markov chain is used to calculate the probability that the bridge or bridge element will be in a particular condition state at a given time. The number of condition states (deterioration levels) in the Markov process is finite (5 condition states are considered in this case). In addition, it should be assumed that the transition from better to worse condition state is operated for no more than one level/rating in the period between two consecutive inspections. Although it sometimes happens that the rating of a bridge or bridge element changes for more than one level in the period between two consecutive bridge inspections, these cases are quite rare and, in such instances, the subjectivity of the inspector's perception during visual inspection can not be excluded [2].

It is assumed that this is a homogenous Markov chain in which transition probabilities p_{ij} are not time-dependent (t_n, t_{n+1}). The process with a finite number r of discrete condition states is considered. In this case, the transition probability matrix P is a square matrix of the r-th order with elements p_{ij}, where the following relation is valid $0 \le p_{ij} \le 1$.

If it is furthermore assumed that during a single discrete period of time (from t_n to t_{n+1}) the process may either remain in the same condition state, or pass on to the next higher condition state, then the transition probability matrix assumes the following form:

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} & 0 & . & 0 \\ 0 & p_{22} & p_{23} & . & 0 \\ 0 & 0 & . & . & 0 \\ 0 & 0 & . & p_{r-1r-1} & p_{r-1r} \\ 0 & 0 & . & 0 & 1 \end{bmatrix} \qquad \begin{matrix} p_{11} + p_{12} = 1 \\ p_{22} + p_{23} = 1 \\ \\ p_{r-1r-1} + p_{r-1r} = 1 \end{matrix} \tag{1}$$

where:

P - transition probability matrix,

p_{ij} - the probability that the process will pass from the condition state i at the moment t_n to the condition state j at the moment $t_{n+1} > t_n$ (i≠j),

p_{ii} - probability that the process will remain in the condition state i in the time period from t_n to t_{n+1}.

The Markov chain theory assumes that the sum of each row of matrix (1) is equal to 1:

$$\sum_{j=1}^{r} p_{ij} = 1 \tag{2}$$

In addition to the transition probability matrix, the initial condition state probability vector must also be known for estimating probability of a process at a given future moment.

$$\mathbf{p}(0) = [p_1(0) \quad p_2(0) \dots p_r(0)] \tag{3}$$

Table 2. Distribution of superstructure condition state ratings for bridges included in the sample, by inspection periods

Period	Reference year	Condition 1	Condition 2	Condition 3	Condition 4	Condition 5
1996 – 2000	2000.	59	33	15	0	0
2001 – 2006	2006.	35	42	25	5	0
2007 - 2012	2012.	26	47	26	8	0

where $p_i(0)$ ($i = 1, \dots r$) denotes probability that the process will be in the condition state i at the initial moment ($t = 0$). In our case, the initial condition state vector is formed based on the latest performed set of bridge inspections.

Knowing the values of both the transition probability matrix P and the initial condition state probability vector, it is now possible to determine the condition state probability vector $p(t_n)$ at any given moment t_n that shows the probability that the process will assume at a given moment t_n one of the condition states r. This vector is expressed via a matrix equation:

$$\mathbf{p}(t_n) = \mathbf{p}(0)\,\mathbf{P}^n \tag{4}$$

where:

$$\mathbf{p}(t_n) = [p_1(t_n) \quad p_2(t_n) \quad p_3(t_n) \dots p_r(t_n)] \tag{5}$$

At that $p_i(t_n)$ (i = 1, ... r) is the probability that the process will be in the condition state i at the moment t_n.

The transition probability matrix is a historic record about the way in which elements, similar in age and type, are deteriorating. In the sample under study, the starting point of the model is in the data on the condition state of superstructure for the sample of 107 bridges from the database, which were prepared as shown in Table 2.

The transition probability matrix assumes the following form:

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} & 0 & 0 & 0 \\ 0 & p_{22} & p_{23} & 0 & 0 \\ 0 & 0 & p_{33} & p_{34} & 0 \\ 0 & 0 & 0 & p_{44} & p_{45} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

where:

$p_{11}, p_{22}, p_{33}, p_{44}$ - probability that the process will remain in the condition state i (i = 1 do 4) during the six-year period in-between the inspections,

$p_{55}=1$ - because the element can not pass from the condition state 5 to any other condition state,

$p_{12}, p_{23}, p_{34}, p_{45}$ - probability that the process will pass into a higher condition state in the same period.

In order to describe by model the continuous nature of the deterioration process, transition probabilities are calculated based on relative frequency of occurrence of events under study. At that, two events can be defined for every bridge in every transition period: the first event is defined as "no change of condition state" and the other as "transition to a higher (worse) condition state". As the condition state measurement data are available for three discrete time points (Table 2), i.e. for two transition periods, it would be possible to present 107 x 2 = 214 events or transitions. Transition probabilities are calculated from relative frequencies of events (Table 3).

Table 3. Calculation of transition probabilities. Values marked in the model with red are values corrected based on empirical data

Transition from one condition to another		Frequency	Total number of events	Probabilities	
				P_{11}	0,65
1	2	94-61=33		P_{12}	0,35
2	2	(42+47)-33=56	33+42=75	P_{22}	0,75
2	3	75-56=19		P_{23}	0,25
3	3	(25+26)-19=32	15+25=40	P_{33}	0,80
3	4	40-32=8		P_{34}	0,20
4	4	(8+5)-8=5	0+5=5	P_{44}	1
4	5	5-5=0		P_{45}	0
5	5	0	0	P_{55}	1

For instance, according to superstructure condition state data for the sample of 107 bridges, there are f_{11} = 61 events in which the condition state 1 will remain unchanged and f_{12} = 33 events involving transition to a higher (worse) condition state 2. The corresponding relative frequencies, i.e. transition probabilities are p_{11} = 61/94 = 0,648936 and p_{12} = 33/94 = 0,351064. An analogous procedure is used to calculate other transition probabilities (Figure 5).

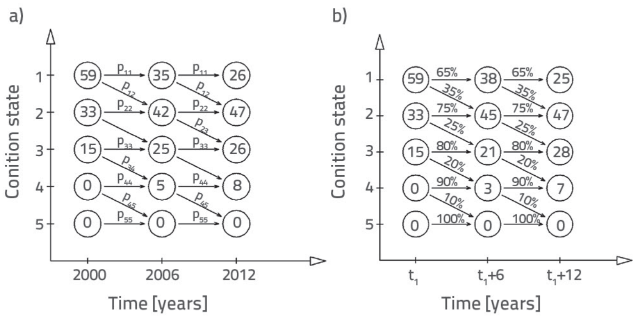

Figure 5. a) Basis for calculation of transition probabilities: presentation of three ratings sets, according to Tab 2, b) Markov chain model with theoretical description of deterioration shown on sets given in Figure 5a

It can be seen from the above data that, in the period between two inspections, the heavy damage (rating 4) never passes into the rating which calls for traffic restrictions (rating 5). In reality, this case does occur, and the lack of observation results from the fact that repairs are made quite rapidly when condition state 4 is detected (in this case "rapidly" means within six years). The experience shows that, in the period between the inspections, at least 10 % of bridges pass from condition state 4 to condition state 5, and the matrix corrected by this empirical information is shown in Figure 5b.

With transition probability matrix coefficients calculated in this way, the initial condition state vector (7), determined using the last available set of results from 2012, is used for estimating future deterioration trends.

$$\mathbf{p}(0) = [0{,}24 \quad 0{,}44 \quad 0{,}24 \quad 0{,}08 \quad 0{,}0] \tag{7}$$

The prognosis of the bridge element condition state in six year intervals, presented in Figure 6, can be interpreted as follows: 24 years after the last bridge condition state measurement, without any repairs and interventions, only 5 out of 26 bridges will remain in condition state 1. The number of bridges in condition state 2 will reduce from 47 to 27, while the number of bridges in condition state 3 will increase from 26 to 40. The number of bridges in condition state 4 will increase by 10 in 12 years. During initial bridge inspection, there are no bridges in the worst condition state (condition 5). However, if no repairs are made, there will be 6 of such bridges at the end of the period under study.

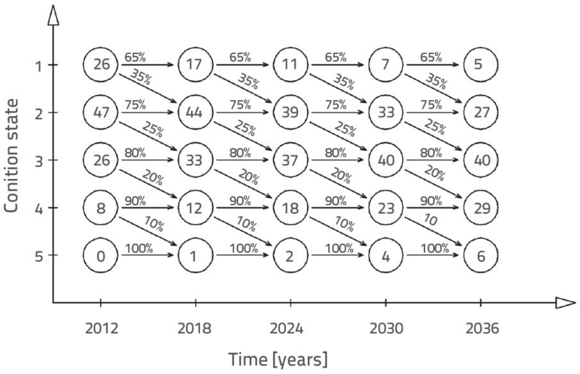

Figure 6. Prognosis of bridge distribution by condition state over the future six—year intervals, based on Markov chain model

3.4. Markov process model

Basic theoretical assumptions of the stochastic model based on Markov model of the real physical process are presented in [4]. Transition probabilities $p_{ij}(t)$ derived from some assumptions, e.g. about the physical bridge-deterioration process (or processes), must be known to determine probability of the condition state j at a given moment t. As these are functions, it is normally a very difficult problem which, in many cases, can not be solved in practice. A favourable circumstance is the fact that the Markov principle ("the future is influenced by the present time, rather than by past") enables determination of these functions on the basis of their "infinitesimal properties". This means that the theory of homogenous Markov processes guarantees that the

Table 4. Data on the inspection of five bridges with the illustration on the reasoning used to define the time bridges remain in the n-th condition state

Bridge name	Inspection results								Conclusion		
	Inspection 1		Inspection 2		Inspection 3		Inspection 4				
	Age [years]	Rating	Age [years]	Rating	Age [years]	Rating	Age [years]]	Rating	Condi-tion	>n [god]	<n [god]
Frigis Overpass	18	1	23	2	29	2	-		1	18	23
									2	-	-
Klajnova Overpass	25	3	31	4	37	4	-		3	-	31
										6	-
Sigetac Ludbreški	10	1	12	1	15	1	20	2	1	15	20
Drnje	68	1	73	3	79	3	-		1	68	73
									2	-	5
									3	6	-
Velika Ves	28	2	31	2	33	3	40	4	2	3	33
									3	-	9
					Average time the bridge stays in the n-th condition				1	33	39
									2	5	19
									3	6	20
									4	6	-

functions will be satisfactory for some differential equations in which the following values appear as coefficients:

$$\lambda_{ij} = \frac{dp_{ij}(0)}{dt}, \quad i,j = 1,2,\ldots,r \tag{8}$$

where $\frac{dp_{ij}(0)}{dt}$ is the derivation of the function $p_{ij}(t)$ along the variable t, for $t = 0$.

If the value Z_i ,is introduced, which stands for the time to process remains in the i-th condition state, the theory of homogeneous Markov processes shows that Z_i (i = 1, ... r) is the random variable including the exponential distribution with the parameter $-\lambda_{ii} = \lambda_i \geq 0$. Consequently, the expected time the process remains in the condition state *i* amounts to:

$$E[Z_i] = \frac{1}{\lambda_i}, \quad i = 1,\ldots,r \tag{9}$$

The input parameter for model is the time during which bridge elements remain in a given condition state. This information is not derived directly from the database, which contains condition state measurements made in a given moment. That is why the procedure based on mathematical logic principle has been developed to deduce or to estimate the time during which the element stays in any of these condition states. This procedure is presented in Table 4 on an example of several bridges. As values λ_i are directly influenced by the time the elements stay in a given condition state, it is important to note that the sample may contain only those bridges or bridge elements that have never been significantly repaired. If this were not the case, the conclusions about the time the elements stay in a given condition state would not be accurate.

The time the bridge elements remain in each of typical condition states Z_i (i = 1, ... 5) were defined by a detailed analysis of the above described statistical samples. The graphical presentation of these values results in the deterioration trajectory that is shown in Figure 7.

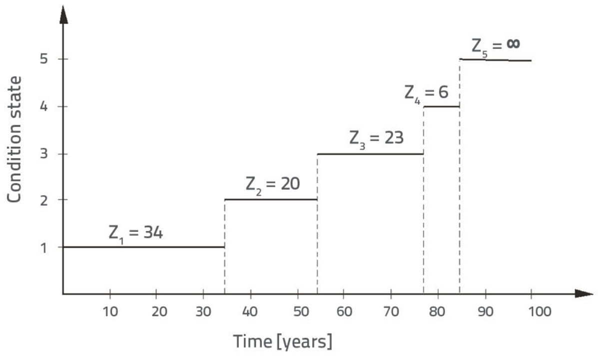

Figure 7. **Deterioration trajectory (mean values from Table 4) prepared for developing the model for a concrete bridge superstructure element**

It is important to note that the presented procedure provides a time frame within which, according to statistical data, the bridge remains in a particular condition state. The final selection of the time frame to be used in modelling (process trajectory) can be improved by comparing the statistical model with the physical model, i.e. with one of the models used for service life calculation according to [12]. The testing needed to calibrate the physical model was conducted on a smaller number of bridges situated along the national road network, but it can be used for the adjustment of statistical model parameters.

Formulas for determining probability of each of the five condition states at any moment *t* are calculated using the theory presented in [4]. The initial probability vector corresponds to the vector used for the model with Markov chain (7).

For the known initial condition state and for the known time the element remains in a condition state (and these are the model parameters), the calculation continues by determining probabilities that a bridge element will be at a certain moment of time in one of the mentioned condition states. The graphical presentation of condition states, given in Figure 8, can be interpreted as follows: the condition state 2 will most probably apply during 10 years since the start of the monitoring process, the condition state 3 from the 10th to the 23rd year of monitoring, and the condition state 5 will most probably apply in the ensuing period. The condition state 4 is never the most probable one but it stagnates at about 8 % during the period of some 40 years.

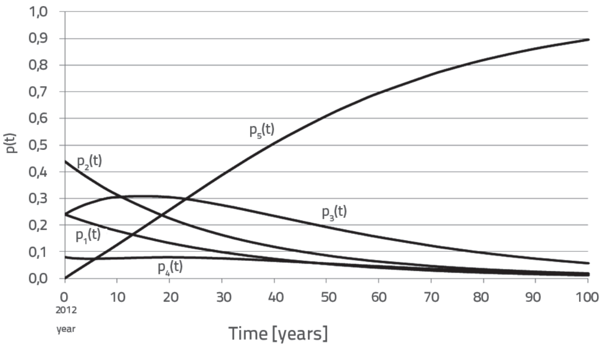

Figure 8. **Condition state probability for a concrete bridge superstructure element: forecast based on the data gathered during inspection carried out from 1998 to 2012, using the Markov process model**

For planning maintenance activities, it is important to note that the deterioration model set in this way can answer the following questions:

1. What is the probability that the element will be in condition state 5 after 10 years, i.e. what is the percentage of bridges on which repairs involving interruption of traffic will have to be undertaken if significant repairs are not made by the year 2022? The value can directly be calculated or read from the graph $p_5(10) = 0{,}1257 \approx 13\ \%$

2. What is the probability p(20) that the element will not be in the condition state "damaged" in the year 2032?

The probability that the element will be in condition state 1 ($p_1(20)$), 2 ($p_2(20)$), 3 ($p_3(20)$) OR 4 ($p_4(20)$) IN 2032 is:

$$p_1(20) + p_2(20) + p_3(20) + p_4(20) = 1 - p_5(20) = 1 - 0{,}2563 = 0{,}7437$$

The required probability expressed in percentage amounts to 74.4 % ≈ 74 %, and so it can be stated that 74 % of elements will remain undamaged.

3. At which point in time, i.e. after how many years will the probability of the elements attaining the worst condition state (5, damaged) exceed 50 %?
At the graph of the function $p_5(t)$ we can see that this will happen after approximately 40 years.

4. What is the expected condition state of an element after 30 years of bridge use without any maintenance?
Besides the probability of individual condition states, the expected deterioration of the structure at a given moment t can be written as follows for the model with five condition states:
$E[S(t)] = 1 \cdot p_1(t) + 2 \cdot p_2(t) + 3 \cdot p_3(t) + 4 \cdot p_4(t) + 5 \cdot p_5(t)$

By inserting concrete values in the formula we obtain:
$$E[S(t)] = 1 \cdot 0{,}099314 + 2 \cdot 0{,}163553 + 3 \cdot 0{,}27465 + 4 \cdot 0{,}075779735 + 5 \cdot 0{,}386703458 = 3{,}49$$

It can be noted that the expected condition state after thirty years will be approximately 3.

4. Discussion: advantages and drawbacks of presented models

The objective of this research was to create a process progress model that could be used for forecasting future deterioration of all similar structures or structural elements within the network. Two questions should be put with regard to model analysis:
1. What should be done to prepare the existing data for analysis?
2. What questions are answered through modelling results?

A regression model is prepared directly from existing data (ordered pairs: bridge age – condition state assessment) and is in this respect the least demanding model. However, the result shows that this tool can not be used for condition state forecasting purposes, as it only points to deterioration trends of individual elements. In fact, the function graph shown in Figure 3 is near the rating 2 almost throughout the service life of the bridge. By creating graphs for all elements, we would possibly obtain trends that could be compared by saying that one elements deteriorates faster than the other. The result would not be significantly changed by modification of the regression function (e.g. by selecting an exponential function instead of the straight line), and so this model has been rated as the worst option.

The model based on the *Markov chain theory* is the most often used in international practice, but it requires a continuous set of measurements in regular intervals, each no longer than two years. Inspections are not always conducted in equal intervals nor frequently enough to make the model using the Markov chains an optimum solution for the data that are at our disposal. The data used in our research are appropriate for a model involving reduction to a reference year, i.e. if the inspection were conducted in 2004 the rating would be transferred to the reference year 2006, which does not need to be correct. Thus, principal model shortcomings arise from the work on the discrete time scale (6-year intervals) and from difficulties regarding correction of parameters based on empirical findings. In this concrete case, the model would be acceptable it there were a set of, for instance, 5 condition state measurements spaced at 2 year intervals, especially as six-year intervals between predicted condition states are not sufficient for planning (we need forecast for the following year and for four-year periods). In addition, the notion of transition probabilities is not close to the engineering way of thinking about deterioration of bridges.

The model in which *homogeneous Markov processes* are used is based on the data about the time the bridge element will remain in a particular condition state. Estimates about how long something will last are close to the engineering way of thinking (e.g. if we do not repair it, the bridge will last for the n number of years). This model also requires some preparation of data, i.e. statistical determination of the time the process will remain in a particular condition states, but this problem has successfully been solved by the presented algorithm.

Compared to the previous two, this model is the most appropriate as the period the bridge element remains in a particular condition state can be estimated through identification of processes that cause damage to such element, and this by using the physical deterioration model. The efficiency of the model that is used to estimate the service life of reinforced-concrete structures has been investigated on the example of our bridges [12], which demonstrates that it can be used for correction of statistically determined deterioration parameters. By linking the bridge rating with an active process, it is possible to define more accurately the moment when a bridge element passes from one condition state into another, which enables a more exact measurement of bridge condition state. For reinforcement corrosion, this can be presented as follows: condition state 2 – start of corrosion, condition state 3 – active corrosion, condition state 4 – visible corrosion and damage to reinforcement. Calculation of the number of years during which the structure will remain in one of the condition states enables better argumentation at the planning stage, as these condition states are related to known repair methods (establishing the link between the condition state and typical cost estimate is the next step in the elaboration of the procedure).

All three models presented in the paper are statistical models. They are based on the analysis of data on the numerically expressed condition state of a bridge element selected from the database of visual inspections conducted, in accordance with an appropriate methodology, by bridge engineers from the company Hrvatske ceste in the period from 2000 to 2012. Several detailed bridge inspections were conducted in the same period in our country and internationally in order to define the type of process that demolishes the bridge, and the rate at which this process is advancing. Considering Croatia's specific condition states, particularly valuable results are those that have been obtained in the study of corrosion of reinforcement due to chloride penetration into concrete.

Further research should be oriented toward unification and analysis of data on deterioration processes affecting our bridges, and toward treatment of such data so that they can be linked with bridge ratings. In the concrete case, it is known that investigations works preceding remedial activities were actually conducted on several bridges. The findings should be related to condition state ratings that are defined by visual inspections, so as to correct current estimates about duration of individual bridge condition states and related processes. After that, the notion of condition state should be linked with typical repairs, and this separately for each bridge element. The final objective is to create a tool that will enable us to forecast the need for repair work through technical and financial indicators, based on information from the existing database, without making significant changes to the well established methodology currently used by HC.

5. Conclusion

Three concrete bridge superstructure deterioration models were established based on the existing bridge inspection database, i.e. based on inspections conducted from 2000 to 2012, in order to determine which one of these models is the most appropriate for maintenance planning.

All these process-based models first point to the questionable validity of input results, as manifested through excessively long time structural elements are assumed to be in condition states implying damage, but not requiring repair work. Based on previous analyses of visual inspection procedures [2] and some analyses of remedial activities [13], it can be concluded that the models are unable to objectively present the progress of deterioration processes. This is attributed to the assessors' reluctance to give higher rating until it becomes fully evident that the element or structure is damaged to such extent that the safe operation of traffic is put in question.

If the possibility of combining statistical parameters with empirical data is taken as the criterion for selecting an appropriate deterioration model, then the best bridge maintenance planning model is the one based on homogenous Markov processes. The model based on Markov chains, which is currently most often used on a worldwide scale, would have an advantage over the others if principal inspections were conducted more often, and if we had longer series of inspection results.

In any case, the basic model should be based on the statistical treatment of data on the registered bridge condition state. The developed model should be corrected by information on physical deterioration processes, as gathered through appropriate analyses. In this case, the design time the element remains in a given condition state, which is either calculated or estimate, serves as the information that can be used for correction of the deterioration parameter as calculated according to statistical data. Considering a relatively small number of available data and their unreliability, it is highly advisable to use all available sources of data, i.e. statistical data, theoretical deterioration models confirmed by laboratory testing, and practical experience of engineers.

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