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Construction process optimisation – review of methods, tools and applications

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Subject review

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Construction process optimisation – review of methods, tools and applications

A review of current heuristic techniques and mathematical programming methods is presented in the paper. Modern optimisation modelling tools are presented, common construction optimisation problems are described, and an overview of their recent application is given. It is also stressed that there is ample room for further research on active BIM-based optimisation models, aimed at better management of construction processes. Accelerated development of decision-making models, which combine optimisation methods and information systems, can be clearly predicted for the near future.

Key words:

BIM, construction production, modelling, optimisation methods, process optimisation

Pregledni rad

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Optimizacija građevinskih procesa – metode, alati i primjena

U radu je dan pregled suvremenih heurističkih tehnika i metoda matematičkog programiranja. Predstavljani su moderni alati za optimizacijsko modeliranje, izlažu se najčešći problemi optimizacije u građevinarstvu i daje pregled njihovih nedavnih aplikacija. Rad je istaknuo prostor istraživanja na aktivnim modelima optimizacije, koji su podržani BIM-om s namjerom boljeg upravljanja građevinskim procesima. Ubrzani razvoj modela odlučivanja, koji kombiniraju optimizacijske metode i informacijske sustave, jasno se može predvidjeti za blisku budućnost.

Ključne riječi:

BIM, građevinska proizvodnja, modeliranje, optimizacijske metode, optimizacija procesa

Übersichtsarbeit

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Optimierung von Bauprozessen – Methoden, Werkzeuge und Anwendung

In der Abhandlung wird eine Übersicht über die modernen heuristischen Techniken und Methoden der mathematischen Programmierung dargelegt. Vorgestellt werden moderne Werkzeuge für die optimierte Modellierung, es werden die häufigsten Probleme der Optimierung im Bauwesen dargelegt und es wird eine Übersicht ihrer aktuellen Anwendungen gegeben. Die Abhandlung betont den Untersuchungsbereich aktiver Modelle der Optimierung, die durch BIM unterstützt werden, mit dem Ziel eines besseren Bauprozessmanagements. Eine beschleunigte Entwicklung der Entscheidungsmodelle, welche die Optimierungsmethode und die Informationssysteme kombinieren, kann für die nahe Zukunft klar vorausgesehen werden.

Schlüsselwörter:

BIM, Bauproduktion, Modellierung, Optimierungsmethode, Optimierungsprozess

1. Introduction

Optimisation has been present for quite a long time in industrial production. However, it can be noted that optimisation has once again become the subject of highly intensive worldwide research, following a huge theoretical growth of this field in the period between 1950s and 1970s. Reasons why this field of science is still so propulsive can be found in an increased common awareness about limited availability of resources, what happens to be the main trigger that pushed the optimisation forward in the list of public's criteria. Additional motivators are wide availability of computational packages and steadily growing hardware capacities. Hence, the almost forgotten mathematical models are now being revised, expanded, modified and verified through their use in solving actual optimisation problems in industrial production.

The quantitative support to decision making processes in industry is especially highlighted as being of crucial significance in modern management trends. However, the specificity of managing construction projects is their stochastic and dynamic environment, despite common repetitive operations, in which optimisation constitutes one of the phases. At this point, the flow of suitable information is highly important for performing optimisation in construction processes and, therefore, it is necessary to identify which of the available solution techniques is the most appropriate for a particular problem.

Although construction industry has always been project-oriented [1], as well as affected by inefficiency and ineffectiveness [2], limited attention has been paid in practice to mathematically-based approaches for processes optimisation. Therefore, better awareness about contemporary techniques for processes optimisation can be generally beneficial to all decision-makers in construction industry, especially to project managers and experts for production preparation.

It is worth mentioning that improvement of business processes can also be achieved in industry through enterprise resource planning (ERP) systems. Various ERP systems for manufacturing and trading companies are known and some of them have been supported by rationalized workflows [3] and schedules [4]. Subject reviews have also been presented in literature [5] and some authors, e.g. [6], introduced a systematic approach for ERP implementation in construction sector. It has often been noted that ERP systems can be good vehicles for business process reengineering and introduction of IT systems [7]. Nevertheless, this article does not attempt to provide a review of integration and interoperability between different information systems, e.g. ERP and building information modelling (BIM), since literature on that subject can be found in relevant databases, and some recent contributions have even indicated that there still exists a vast scarcity of knowledge in the mentioned field, see reference However, BIM is steadily being introduced in construction industry, and it is necessary to review current state in the sphere of process optimisation, since both concepts can be operatively connected in order to improve

business performance. To the best of our knowledge, such a review has not as yet been made. Authors [8] stated that BIM has evolved and that *The construction community is seeing a shift from the 3D or visualization aspect of BIM to workflow-specific tools that are being directly applied to solve real-world problems, such as installation verification, sequencing, and estimating. The industry dialogue is now moving to a general questioning of how we optimize the effective capture, analysis, and dissemination of information in real time to make projects more successful.* Therefore, this paper gives an overview of recent achievements in the mentioned fields, and provides some findings that are intended to fill gaps in literature and open possibilities for further research.

Considering the great variety of problems encountered, and in the light of previous literature reviews concerning specific solution methods and modelling tools, the authors present a review of those concerning the optimisation of construction production in order to provide the expert and academic communities with current information relating to these areas.

2. Optimisation methods

Optimisation methods can be used for solving a wide range of different engineering problems. Even though optimisation problems may come from various fields and different systems, they can basically be formulated in a surprisingly similar way. Generally, an optimisation problem (OP) can be expressed in the following way:

$$\begin{aligned} &\text{minimize } f(\mathbf{x}), \\ &\text{subject to: } \mathbf{h}(\mathbf{x}) = \mathbf{0} \text{ and } \mathbf{g}(\mathbf{x}) \leq \mathbf{0} \end{aligned}$$

where $f(\mathbf{x})$ is the objective function to be minimized over the vector of decision variables \mathbf{x} , while $\mathbf{h}(\mathbf{x}) = \mathbf{0}$ represents equality constraints and $\mathbf{g}(\mathbf{x}) \leq \mathbf{0}$ denotes inequality constraints.

The objective function defines the criterion for selecting an optimum solution, while the constraints determine boundaries delimiting the space of all feasible solutions. It should be noted here that the objective function may also be maximized over the feasible space if necessary. As to decision variables, they are usually calculated between their lower and upper bounds, $\mathbf{x}^{\text{lo}} \leq \mathbf{x} \leq \mathbf{x}^{\text{up}}$, and they can be continuous, $\mathbf{x} \in \mathbf{R}$, where \mathbf{R} is the set of real numbers; or integer, $\mathbf{x} \in \mathbf{Z}$, where \mathbf{Z} is the set of integers. Integer variables can also appear as binary decision variables, i.e. $\mathbf{x} \in \{0,1\}^m$.

An appropriate method for solving a specific optimisation problem should be selected attentively in order to obtain valuable results. Before selecting a method of solution, the optimisation problem should be analysed from the standpoint of its functions, constraints, and decision variables. Solution approaches for single-criteria optimisation problems can be roughly divided into two main groups, i.e. heuristic methods and mathematical programming methods.

Heuristic techniques can be used to solve a wide variety of optimisation tasks and their main advantage is that most of

these techniques converge reasonably fast, and can handle problems that contain non-differentiable functions. However, at the end of the search, heuristic algorithms often offer only approximately optimal, i.e. sub-optimal solutions. Nevertheless, heuristic methods have been proven suitable for solving a variety of optimisation problems in civil engineering, and the most frequently used ones are: direct search (DS) [9], evolution strategies (ES) [10, 11] and genetic algorithms (GA) [12,13], tabu search (TS) [14], simulated annealing (SA) [15], neural networks (NN) [16], ant colony optimisation (ACO) [17], particle swarm optimisation (PSO) [18], differential evolution (DE) [19], and harmony search (HS) [20]. As extensions to main heuristic methods, the meta- and hyper-heuristic stochastic techniques are mostly inclined to bio-inspired computing algorithms like bacterial foraging optimisation (BFO) [21], cuckoo search (CS) [22], artificial bee colony (ABC) [23], firefly algorithm (FA) [24], bat algorithm (BA) [25], flower pollination algorithm (FPA) [26], artificial plant optimisation (APO) [27], wolf search algorithm (WSA) [28], etc. It should be noted here that various hybrid combinations of the aforesaid techniques have also been reported in literature.

Mathematical programming methods have also been widely recognized as advantageous tools for optimisation in civil engineering (examples supported with references will be given in sections 4 and 5). The major benefit of mathematical programming methods is that they provide an expected exact optimum result, although the search process itself can, in some cases, require longer amount of time. As is commonly known, slower convergence of results is typical in the case of optimisation of highly combinatorial, discrete, nonlinear and, particularly, nonconvex problems. In general, the field of mathematical programming covers linear programming (LP), nonlinear programming (NLP), mixed-integer linear programming (MILP) and mixed-integer nonlinear programming (MINLP) methods.

LP is a well developed area of mathematical programming. Solution techniques of LP can generally be applied to various optimisation problems, which include linear objective functions and constraints with continuous variables. Well known simplex algorithm [29] is presumably the most commonly used method for solving LP problems. However, it should be emphasized that the interior point method [30] is often found to be more efficient when it comes to large-scale LP tasks.

When the continuous optimisation problem contains nonlinear functions, NLP approach is required to reach the solution. Many engineering tasks can be translated into NLP problems. A suitable method for solving a particular NLP problem should be selected by considering its size and characteristics of nonlinearity. Currently, there is no standard method that can equally well solve all types of NLP problems. However, as there are many efficient algorithms, an appropriate one can be selected to solve a particular NLP problem. At this point, the generalized reduced gradient method [31], augmented Lagrangian method [32, 33], and successive quadratic programming [34], can probably be mentioned as the most frequently used NLP methods.

MILP is an extension of the LP approach. While LP algorithms can be used to solve continuous linear problems, MILP methods can handle linear optimisation tasks with continuous and discrete decision variables. In MILP problems, discrete variables can be declared as integer variables or as binary 0-1 variables. Standard method for MILP optimisation is the branch and bound method [35] although the cutting plane method [36], as well as the branch and cut method [37], can also be efficiently applied in a number of cases.

MINLP methods are necessary for discrete optimisation problems that include nonlinear terms in their formulation. Although high quality exact solutions can be expected from the state-of-the-art MINLP methods, it should be noted here that the field of nonlinear (discrete) optimisation is highly complex and that it has not yet reached the level of maturity attained by linear (continuous) optimisation. Nevertheless, a number of efficient MINLP methods are available for solving nonlinear discrete optimisation problems in civil engineering, such as the generalized Benders decomposition [38], nonlinear branch and bound method [39], feasibility technique [40], sequential linear discrete programming [41], extended cutting plane method [42], augmented penalty/outer-approximation/equality-relaxation algorithm [43], branch and reduce method [44], mixed-integer alpha BB algorithm [45], hybrid algorithm [46], and others.

3. Optimisation modelling tools

A variety of commercial software for computer-based modelling of optimisation problems are currently available. Syntaxes of algebraic modelling languages are often flexible and allow formulation of large-scale compact optimisation models via indexing. In this way, algebraic modelling languages, such as AIMMS [47], AMPL [48], CAMPS [49], GAMS [50], LINGO [51], LPL [52], MPL [53], OPL [54] and UIMP [55], may be applied for complex and unique optimisation problems, which may need several revisions before an accurate model is established. They are particularly usable in case of considerable number of constraints of the same type that follow a similar pattern. In particular, the algebraic modelling language can simultaneously formulate all constraints of the same type by simultaneously handling decision variables of each type. The application of the algebraic modelling language accelerates numerous model management activities, such as transforming the data into model parameters, modifying the model, accessing the data, and analysing model results.

Spreadsheets are also very popular tools for modelling optimisation problems. Microsoft's computer package Excel (Microsoft Visual Basic), with add-ins like Evolver [56], Solver [57], or What'sBest [58], as well as WinQSB [59], can be defined as spreadsheet based computer programs that are often used for modelling optimisation problems. Spreadsheets are user-friendly tools and enable optimisation model formulation within a familiar software environment. On the other hand, optimisation models generated by spreadsheet software are less transparent than

the ones formulated using the algebraic modelling languages. Tabular entry of model entities is also more time consuming, and so the spreadsheet-oriented modelling software is normally used for creating small- and medium-sized models with a reasonable number of parameters. There are also specialized stand-alone software programs including *inter alia* Gurobi [60], TORA [61] and LIONSolver [62].

Interactive computer languages for numeric and symbolic calculations can also be applied for optimisation modelling. Software packages such as Mathematica [63] and Matlab [64] can also be mentioned as interactive computer languages that are widely used for optimisation modelling, although that is not their main purpose. Both of these software packages provide an interactive environment for data structuring, and for modelling and solving optimisation problems, while also enabling presentation of results in the form of computer printouts and charts.

Clear signs have recently emerged about the growing interest among researchers in the use of internet applications for optimisation. One of the ways of performing optimisation over the internet is to use open programs located at the NEOS

server [65]. At this server, it is also possible to find links to many commercial optimisation programs, manuals for their use, test results, and publications/journals with recent optimisation research data, etc.

4. Use of optimisation in construction processes

4.1. General forms of optimisation problems common to construction processes

This section provides a brief overview of origins and definitions of some well-known optimisation problems that can be recognized as most common in construction processes. Their original name, basic description and objectives, as well as their origin, are shown in alphabetical order in Table 1. Optimisation problems are grouped by their main optimisation objectives, and they form two basic domains of tasks common to construction processes:

- a) optimisation of resources
- b) optimisation of layout and route.

Table 1. Optimisation problems common to construction processes

Domain	Optimisation problem	Problem description	Origin
Resource oriented optimisation problems	Assignment problem (AP).	AP deals with the question of how to assign a given number of tasks to a given number of agents, where each agent can perform any tasks and, at that, this agent creates a cost that changes depending on the task, and all tasks must be completed, provided that only one agent can be assigned a single task, and the total costs must be minimised.	[66]
	Project scheduling problem (PSP).	PSP represents a wide set of problems, which are commonly summarized by one dominant objective function for minimization of total project costs. If expanded with objective functions for minimization of project time and costs, they are known as time-cost trade-off problems (TCTO), and, if including also the optimisation of other resources, they are identified as time-cost-resource optimisation problems (TCRO).	[67]
	Transport problem (TP).	TP is defined as a program for solving transport of goods from multiple sources to multiple destinations with an objective function aimed at minimising transport costs. TP is generally a material network-flow optimisation problem. From the viewpoint of TP formulation, the transport costs, supply and demand quantities, are often input parameters while transporting flows represent decision variables.	[68]
Layout and route oriented optimisation problems	Arc routing problem (ARP).	ARP is a connected graph constructed of two sets of points (origins and destinations) for which it is necessary to find the closed route that visits each destination point at least once, or to determine that such a route does not exist.	[69, 70]
	Capacitated arc routing problem (CARP).	CARP has the objective to find a number of routes such that each arc with positive demand is serviced by exactly one vehicle and, at that, the sum of demand of those arcs, serviced by each vehicle, should not exceed given capacity and the total cost of the routes is minimized.	[71]
	Chinese postman problem (CPP).	CPP is defined by the connected undirected graph with the known distance matrix, and the problem is to find a route that passes through each point of the graph at least once in the shortest possible way.	[72]
	Traveling salesman problem (TSP).	The TSP is defined as an optimisation problem of the salesman who needs to travel from home location to each location specified on the list and, after executing all visits, to return to home location while taking into account the objective of the shortest total route or the minimum total travel time.	[73]
	Vehicle routing problem (VRP).	VRP is defined as a problem of how to optimally route a fleet of identical vehicles from a central point to supply destinations with known demands subject to vehicle capacity constraints.	[74]

Table 2. Resource oriented optimisation problems applied in construction

Period	Description of applied problem	Methods, models and tools	Main findings and conclusions	Origin
1997 - 2000	TCTO problem for an eighteen-activity construction project.	Authors developed a new algorithm by combining genetic algorithm (GA) and Pareto front approach.	The presented new algorithm has proven to be efficient and accurate in solving the addressed problem by searching only a small fraction of the total search space. Furthermore, the authors developed a computer program TCGA that uses the MS Excel program, and automates the execution of the proposed algorithm.	[75]
	TCTO problem involving generation of sixteen construction projects each consisting of eleven activities.	Machine learning genetic algorithm system (MLGAS).	Previous analytical techniques were known to limit the usage of GAs as it was necessary to manually enter data for the time-cost curves and this in linear form only. MLGAS overcomes these limitations by incorporating machine learning via GAs.	[76]
	Multi-objective optimisation of resource allocation and levelling based on a case study involving twenty activities and six resources.	Genetic algorithm was applied by means of MS Visual Basic (VBA) programming language.	Main contributions of the mentioned approach are: effective improvement to resource allocation heuristics using random activity priorities; practical modification to resource levelling heuristics using a double-moment approach; and multi-objective optimisation of both resource allocation and levelling using GAs.	[77]
	Resource levelling problem on a nine-activity construction project with multiple resources, aimed at minimising resource utilization variation within a fixed duration project.	GA (GARLS) based resource levelling system.	GARLS model does not necessarily need to commit to any specific heuristic rule; hence, it is more flexible for solving complex levelling and scheduling problems. GARLS provides several feasible or near-optimal solutions that may assist in project decision-making. In this study, the authors successfully used a variety of software programs for GARLS implementation (i.e. MS Project, MS Access, MS Excel, and VBA). As a recommendation for further development, the authors suggest modification of the proposed model to include TCTO problem solving as well.	[78]
2001 - 2010	Repetitive scheduling problem with shareable resource constraints in case of precast production.	GA-based resource-constrained repetitive scheduling model, using the VBA programming language.	The authors point out that their model incorporates an efficient computational technique for resource allocation and a more suitable way of modelling the resource sharing in repetitive scheduling, compared to the linear scheduling technique. GA-based model does not have to commit to any particular heuristic rules and thus is more flexible. In addition, the GA-based repetitive scheduling model can explore and use several near-optimal solutions, which are normally not available using conventional repetitive scheduling techniques.	[79]
	Repetitive scheduling problem for a project consisting of four similar sections or units, and each includes repetitive activities with finish to start connections without time lags.	Automated model based on dynamic programming formulation.	It was concluded that, since the model is automated, it alleviates the need for the user to provide a set of interruption vectors in an arbitrary manner prior to scheduling. In addition, it significantly reduces the number of interruption vectors in a rational manner, making the optimisation process feasible; it also enables generation of an optimum solution.	[80]
	Resource scheduling and TCTO problem for a twelve-activity construction project with the assumption that there are no limitations on precedence relationships between succeeding activities	Resource levelling using the augmented Lagrangian GA model.	The suggested model allows any linear or nonlinear function for the presentation of cost-duration and resource duration relationships. The model can also handle a wide range of project sizes including large construction projects involving a large number of activities. The model was efficiently implemented in FORTRAN programming language in order to solve resource scheduling problems on several construction projects.	[81]
	Multi-objective TCTO problem that takes into account adaptive weights among optimisation criteria on a case study consisting of seven activities.	GA based model with a modified adaptive weight approach.	Authors note that this approach provides GAs with greater freedom during search in the multi-objective space, which overcomes drawbacks of the single objective TCTO. However, because the model uses GAs in terms of a search engine, its randomness could affect reliability of results.	[82]
	Resource-constrained PSP (RCPPSP) aimed at minimizing project duration.	Metaheuristic PSO based method.	Based on computational analyses, authors concluded that the permutation-based PSO outperforms the priority-based PSO, and that the PSO-based methodology has good performance, which is comparable to other metaheuristic methods such as GA and SA in solving the RCPSP.	[83]
	Stochastic multi-objective TCRO problem with the time and cost variables considered as fuzzy.	Stochastic multi-objective optimisation model based on the use of the non-dominated sorting GAs (NSGA-II).	This model improves the previously developed weighting approaches by providing a three-dimensional Pareto front. It also adopts FSs to incorporate the uncertainties in time and direct costs of project activities. The model can also consider different levels of uncertainty by changing the α -cut levels. Authors suggested that further development of the model could be focused on providing the capability of splitting activities in the model.	[84]
	PSP in case of multiple shifts on construction projects, the aim being to minimize project duration, cost, and negative impacts of evening and night shifts.	Optimisation model consists of three modules: i) initialization module for scheduling optimisation computations; ii) scheduling module; and iii) multi-objective GA module	The model was proven capable of evaluating and identifying optimum shift systems for projects and this already in the first iteration. In addition, the model offers an optimum solution with minimisation of project time and costs, and with appropriate distribution of workers into evening and night shifts, where every solution identifies an optimum schedule and multiple shift work plan for each activity. The model also generates optimum plans for distributing labour among competing shifts to minimize negative impacts of labour constraints on project performance.	[85]

Table 2. Resource oriented optimisation problems applied in construction - extension

2011 – 2016	Non-linear quadratic AP (QAP) for tower crane and material supply locations.	MILP model.	Based on a numerical example, authors found that the results gained by MILP are better than the results obtained by GA, with almost 7% less in the total material transport cost. In addition, MILP formulation was found to be more flexible in terms of including additional design constraint sets for modelling actual on-site conditions.	[86]
	Nonlinear discrete TCTO (NDTCTO) problem for a construction project consisting of twenty-nine activities.	MINLP model.	The proposed model is more complex and requires greater analytical/computational effort than the MILP model. However, the authors noted that the advantage of the MINLP-NDTCTO model compared to the MILP model lies in its modelling capabilities. In addition, the proposed model yields the exact optimum NDTCTP solution, while heuristic models calculate approximate optimum solutions.	[87]
	PSP of the project with modular scaffolding.	Multi-objective constrained optimisation model based on the discrete firefly algorithm (DFA).	Results proved credibility of the optimisation model by producing a better solution of workforce allocation enabling proper time and cost balance.	[88]
	TCTO problem for a construction project with a smaller number of project-significant activities.	PSO method for optimizing global critical path diagrams using the MATLAB programming system.	The authors successfully applied PSO method for optimizing realization of construction projects. The proposed model provides good results and presents several advantages compared to methods based on the use of simplex algorithms for linear programming, and other traditional mathematical programming methods.	[89]
	NDTCTO problem for a construction project with a nonconvex function of costs.	MINLP model.	MINLP optimisation model for handling nonconvex dependencies was found to reduce the user effort in dealing with large-size data and updating the model when circumstances under which the project scheduling was done have changed. The use of nonlinear expressions can enable a more compact model formulation as well as a more rapid execution of model management tasks, such as transformation of data into model parameters and model modifications.	[90]
	AP and optimal resource allocation problem on multiple ongoing projects.	Binary multi-objective scenario simulation model.	The model was proven as a useful tool for solving small and medium scale problems. It is adaptable to changes of input data. It also enables comparison of optimum and sub-optimum scenarios with the corresponding output data, and it has thus fulfilled the main expectations.	[91]
	PSP under restricted costs.	MINLP model.	The proposed MINLP model comprises total project costs, generalized precedence relationship constraints, project duration restraints, logical conditions, and cost restrictions.	[92]
	TP for a hot asphalt mixture.	Evolutionary algorithm of the multiple criteria solver (MCS).	Results have proved that the evolutionary algorithm of the MCS is a useful tool for solving the problem of planning transport of hot asphalt mix when the total project realisation time is not limited. The authors point out that the model needs to be modified for solving big problems and for taking into account time and technological relationships between sub processes in the chain.	[93]

Here it should be noted that there are many other variations of optimisation problems, with various objectives and constraints, that would be worthy of consideration. However, they are not presented here due to the limited space available for this paper.

4.2. Overview of recent optimisation applications in construction processes

Due to complex nature of construction problems, and cyclic relations of planning and optimisation processes, it is often difficult to identify only one type of the above presented original optimisation problems, or even to completely differentiate one problem from another. Most optimisation problems in construction appear to be a combination of various original optimisation problems. It is therefore hard to summarize all methods that are used for solving such problems. Nevertheless, in this section, the authors provide an overview of recent and most known (in the majority of cases most cited) applications, methods and models of optimisation problems in construction sector. The overview is given in form of two chronologically structured tables divided by their optimisation orientations: resource oriented construction optimisation problems

(Table 2) and layout and route optimisation problems (Table 3). The layout and route oriented optimisation tasks applied in construction (given in Table 3) are chronologically structured starting from 1995. Since they are not as numerous as the previous set of optimisation problems, they are not divided by longer periods.

5. Optimisation and BIM

5.1. General connections between optimisation and BIM

Optimisation can certainly be considered as potentially beneficial for many areas in construction industry. The basic aim of this section is to present connections established between optimisation and BIM that have been recently reported in reputable scientific literature. Namely, to the best of our knowledge, such a review has not yet been carried out notwithstanding the fact that it concerns a rapidly developing field. Overview of recent achievements in the context of connecting optimisation and BIM is especially needed to identify new perspective topics for research and to fill the literature gap.

Table 3. Layout and route oriented optimisation problems applied in construction

Description of applied problem	Methods, models and tools	Findings and conclusions	Origin
Planning the construction-site access routes for large vehicles.	The route-planning system was achieved using the expert system Nexpert Object and the geographic information system (GIS) Arc/Info in conjunction with the computer-aided design (CAD) package Microstation, Excel spreadsheet, and other custom programs.	Alongside the "what-if" analysis, the authors underline that the user can employ the presented system to select modification parameters, modify selected parameters, and test modification results. In further work, authors suggest developments in the following spheres: model uncertainty and ability to identify areas of risk on potential routes; quantification of user preferences using a fuzzy-logic approach; and representation of strategies and knowledge required for scenario modification in order to facilitate access to site.	[94]
Construction site layout problem (CSLP) aimed at minimising material transport costs.	GA generates an initial population of layouts through a sequence of mutation operations, and evolves population layouts through a sequence of genetic operations aimed at finding an optimum site layout solution.	Authors emphasize that the key feature of the proposed algorithm is that it uses a large number of different GA operators to vary positions of objects around the site. The GA operators are programmed in such order that the chance of finding a feasible position for a selected block is maximized with a function that finds and stores sets of feasible positions for a selected object. Another key feature is that it maintains in each generation the chromosomes representing partial layout solutions. These so-called "bad" chromosomes are kept to help the evolution process get out of local optima.	[95]
Scheduling, resource planning, and cost optimisation problem in the scope of large construction and maintenance programs that involve multiple distributed sites with the objective of finding an optimum set of construction methods and an optimum routing order among sites.	The authors propose a distributed scheduling model (DSM) that uses GAs to determine an optimum set of construction methods and an optimum routing order among sites.	The DSM has proven to be an indispensable additional tool during determination of an appropriate technology and an eligible and feasible list of facilities to be included in municipal construction/maintenance programs. Its benefits include determination of the number of required crews and their detailed work plan, and conduct of sensitivity analysis studies for determining the most proper time to start execution of a construction program; it can also provide a dynamic environment needed to meet constraints and decide on proper corrective actions during execution.	[96]
Logistic and dispatching problems occurring during disaster relief activities. They are characterized by temporary escape route problems, vehicle directing problems, and the multi-commodity dispatch problems.	Meta-heuristic method of ant colony optimisation (ACO), which decomposes the original emergency logistics problem into two sequential phases and iterates between them.	In comparison to the CPLEX solution, the quality solution gained by ACO is achieved within a minute of runtime, which is especially significant in emergency situations involving continuous uncertainty and information dynamism. However, the local search based meta-heuristics, such as tabu search, needs further study to prove its efficiency in solving this problem. It should be noted that introduction of local search into the post-optimisation procedure does not enhance the overall solution efficiency, although some provisional solution quality is improved in the process. Computational results suggest that this decomposition approach may be efficient for other complex combinatorial problems with interdependent decision variables.	[97]
Traffic delays and scheduling problem based on the route-changing behaviour of road users.	Micro-simulation model involving team ACO (TACO) search for a near-optimal scheduling, based on the VISSIM simulation software.	Results indicate that the total traffic delay can significantly be reduced by means of the proposed model. Compared with other mathematical methods, the microscopic simulation requires more computational time, but it is closer to the real-time situation, what makes the prediction and estimation more reliable.	[98]
VRP model for truck mixers travelling over a working day from concrete plant to concrete-demanding customers and vice versa.	Combination of MILP model with generic local search approach using the CPLEX program.	General MILP solver is inadequate for solving real time large-scale problems. In order to solve such problems, the authors combined MILP with the local search approach. The suggested approach is based on the assumption that all input data are available for analysis. Therefore, the authors recommend this approach as a useful tool for baseline planning.	[99]
CSLP.	Integrated simulation system consisting of the intricate search and GAs, based on the Symphony platform.	The proposed simulation system can easily be extended to accommodate more disciplines and strategies in order to produce more advanced outputs. The developed modelling system enables tunnelling work experts to create models, and to experiment with different scenarios without the system developer's instructions.	[100]
Dynamic, multi-objective CSLP with unequal-areas.	Max-Min ant system (MMAS) and modified Pareto-based ACO algorithm. The intuitionistic fuzzy	The performance of CSLP decision-making system is verified on the case study of a residential building. It has proven to be a useful tool for project managers and planners in the design of construction sites characterized by multiple conflicting or congruent objectives. In addition, it helps users to design a construction site layout, including other qualitative factors, such as ease of supervision and control.	[101]
TOPSIS method was used for the evaluation and level selection stages.	The MTCARPTW was transformed into TSP in order to apply a heuristic ACO algorithm.	Results suggest that the proposed approach effectively works with a MTCARPTW containing less than fifty nodes, yielding a good solution within a short computing time. Authors suggest that future studies should explore expanding the present problem into practical-sized networks in order to examine whether the ACO algorithm outperforms all known heuristics on the MTCARPTW, and to determine in what way the algorithm's efficiency is affected by an increase in the types of services.	[102]
CSLP as a tower crane layout problem with the material supply and demand optimisation.	Authors compared relative efficiency of the PBA, PSO, and BA for solving the addressed problem.	Results show that PBA performs better than the other two algorithms. Although the PBA performs well in optimizing location of tower cranes, the algorithm is unable to minimize operating costs of supply in case of multi-dimensional problems.	[103]
Ready-mixed concrete dispatching and delivery problem.	The authors compared robust GA and Column Generation (CG) for solving real problems of different sizes.	Results show that, on an average, CG obtains solutions at 20% lesser cost. However, robust GA converges 40% faster than CG, while the number of unassigned customers is almost the same for both techniques.	[104]
Ready-mixed concrete delivery problem.	The authors presented a scenario simulation model using the Enterprise Dynamics software.	The presented model has proven to be a useful tool for solving discrete small and medium size problems related to the delivery of ready-mixed concrete. It provides an optimum solution and also the set of sub optimal solutions, which improves the decision making process for the planner. However, model's dependence on web maps is a major constraint hindering applicability of the model.	[105]

The BIM paradigm is one of the most promising developments in the architecture, engineering, and construction industries [106]. The most frequently used definition is: *BIM is essentially value creating collaboration through the entire life-cycle of an asset, underpinned by the creation, collation and exchange of shared 3D models and intelligent, structured data attached to them* [107]. Building information model is data-rich, object-oriented, intelligent and parametric representation of physical and functional characteristics transformed into a multi-dimensional digital computer-based model. All characteristics about facilities realised through BIM-based tools are available to the user. BIM applications can be categorized according to their basic focus. Most of BIM-based tools manually transform accurate synthetic data into virtual reality and usually contain 3D geometrical model with other visualized information (i.e. dimensions). This approach can be named as passive BIM because the analytical part of the model is absent. The analysis of structural data, such as an optimum time scheduling and construction site workspace planning, risk assignment control, health and safety control or constructability review, also involves the use of other tools, which must be suitably backed by computer skills and expert knowledge of the user.

Viewed in the strict mathematical sense, BIM as a concept is not an optimisation approach until at least one of the optimisation methods is applied. This puts into perspective the differences between the simulated solution and the optimized one. The main purpose of this section is to place emphasis on recently published BIM-based approaches, operationally named as active BIM approaches. In these cases, models are working with analytical data through algorithms. The following subsections concentrate on only those active BIM applications that employ optimisation techniques and are most frequently identified in construction management literature, i.e. particularly those for APs, CSLPs, and PSPs.

5.2. Active BIM applications

The aim of the following table is to show active BIM applications (i.e. integration of optimisation techniques and BIM) developed for the purpose of generating optimum results. Most frequently identified applications are namely the ones for APs and CSLPs. Table 4 shows contributions in a time-sorted structure. Active BIM applications for PSPs are shown by chronological order in Table 5.

Table 4. Active BIM applications for APs and CSLPs

Year	Description of applied problem	Methods, models and tools	Main findings and conclusions	Origin
2003*	AP for finding optimal positions of temporary facilities in CSLP.	Authors developed a new algorithm by using GA on the CAD-based platform.	The paper concluded that the GA by itself does not ensure an optimum CSLP solution, but may offer a near optimum solution. By minimizing the objective function, the GA accomplishes the complex task of assigning temporary facilities to positions consistent with their respective proximity needs.	[108]
2005*	AP for optimizing location of cranes and facilities.	GA method was used for positioning cranes and facilities.	Significant time savings can be made using the GA model for crane positioning on construction sites. The setting of two types of chromosomes has been found useful for generating the GA model. The first chromosome indicates the crane position code while the second one indicates the number of cranes. Future work can be extended to integrate and hybridize the GA-based model with the 3D visualization technique.	[109]
2010*	CSLP for airport expansion projects.	GA method was used for positioning cranes and facilities.	Significant time savings can be made using the GA model for crane positioning on construction sites. The setting of two types of chromosomes has been found useful for generating the GA model. The first chromosome indicates the crane position code while the second one indicates the number of cranes. Future work can be extended to integrate and hybridize the GA-based model with the 3D visualization technique.	[110]
2014	BIM-based CSLP focusing on actual travel paths.	GA was employed to generate dynamic CSLP model. Autodesk Revit was used as the BIM tool and Microsoft Project was employed for scheduling. The authors developed their own software that reads excel spreadsheets.	Results show that the linear distance based optimisation to minimize the transport distance of site personnel could lead to the generation of sub-optimal layouts, and hence the actual travel distance was used. The method generated layouts that reduced total on-site travel distances by 16.5%. Future research on this model could be oriented toward integration of the 4D construction simulation.	[111]
2015	BIM-based automated CSLP for congested construction sites.	GA was applied to generate dynamic CSLP models. Calculations were based on BIM generated data.	Results show a reduction of approximately 13.5 % in the actual travel distance on construction site compared to conventional methods. The authors concluded that the conventional algorithm, which uses direct distances in the optimisation process, is not sufficient. The model could be extended to include real time construction schedules and material logistics models.	[112]
2015	BIM-based automated CSLP for construction site cost optimisation	GA, BIM and RFID are combined to optimise generation of construction site layouts in real time.	The results show that the proposed system can automatically track temporary facilities, and model available construction site spaces, in order to minimise costs.	[113]

Table 4. Active BIM applications for APs and CSLPs - extension

2014 2015	AP for generating an optimal tower crane layout plan.	A BIM application is used to automatically generate material quantities needed for on-site transport. FA-based optimisation is employed to determine tower-crane locations, as well as material source and destination points.	Results show that less time is needed to create a tower crane layout scheme in comparison with traditional methods, especially when more than one tower crane is used. The method also generates savings in total material transport costs and possible collision costs as, due to visualisation of the process, the workers can easily understand and implement the tower crane layout scheme.	[114] [115]
2015	AP for optimum tower crane positioning and crane type selection.	The analytical hierarchy process (AHP) is utilized as a multi-criteria decision making (MCDM) method for crane type selection. GA is used for defining an optimum crane layout, shortest rail length, and an optimum lifting assignment plan. BIM model is utilized for extracting quantities required for GA optimisation.	Results show that the use of AHP method for crane type selection is quite significant and that it present the highest level of sensitivity by offering the greatest number of critical selection criteria. The presented example shows that the hammer head tower crane is the most sensitive crane type since it has obtained the highest sensitivity coefficients. Authors concluded that the tower cranes BIM model can additionally be developed by adding more functionalities such as enabling advanced spatial query functions, or introducing mobile crane modelling capabilities.	[116]
2016	AP for estimating travel frequencies in CSLP. Objective function minimizes travel costs.	Integration of BIM input data and travel frequency scheduling is presented. The VBA analyser is coded for linking information gathered from both BIM and project schedule. Estimated travel frequencies are set as input data for MILP based CSLP model.	Results show capabilities of the proposed model for automating the travel frequency estimation process. The authors concluded that travel frequency values used from other projects can generate some errors in the optimisation process. It is suggested that further development and improvement in model accuracy be made by collecting own database from more types of projects.	[117]
2016	AP for crane and material supply points positioning, taking into account operating and rental costs.	MILP model for crane location optimisation that minimizes total costs. A case study model is linearized and solved using CPLEX solver.	The tower crane location obtained by the proposed model reveals significant differences when the crane capacity is not taken into account. In this case, approximately 30% reduction in total costs is achieved using the proposed MILP model. The results show that overlooking the crane capacity requirements in the crane location problem may lead to errors as high as 20% when compared to the optimum total costs.	[118]

* Contributions represent CAD-based predecessors of the active BIM-based applications. Their relevance is in the connection between optimisation methods and previous generation of computer-aided tools for AEC industry.

Table 5. Active BIM applications for PSPs

Godina	Opis problema	Metode, modeli i alati	Doprinosi i zaključci	Izvor
2008*	PSP for time-cost integrated schedule.	The object sequencing matrix (OSM) and GA are used. Multi-dimensional CAD is used as model extension.	The methodology of using GA to optimise construction sequences and define crew assignments is presented in six main steps in the paper. The model takes into account constraints regarding resources, workspace and productivity.	[119]
2012	PSP and workspace conflicts problem.	GA-based minimization of workspace conflicts and a 4D CAD system are used.	The proposed study reveals that the workspace model and computed conflicts ratio should be determined prior to the optimisation of workspace conflicts. Thereupon, adjacency of mutual workspaces can be reviewed and schedule conflicts can be detected. Data can be optimized by GA in order to obtain results with minimum duration of conflicts. It is expected that all these functions will be used as basic data for the establishment of an active BIM environment. To the best of our knowledge, the term "Active BIM" is used for in this paper for the very first time.	[120]
2013	PSP for reducing schedule overlaps.	BIM with fuzzy theory function is used for quantifying construction risk. GA is applied for schedule optimisation.	Reduction in schedule overlaps on a construction project with evolution of each GA generation can be visually checked via the presented active BIM system. Results obtained by the proposed GA model reveal that the schedule overlapping rate is reduced by approximately 33 %.	[121]
2013	PSP for scheduling with the minimum overlapping level.	GA and fuzzy-based risk analysis are used.	The authors demonstrate that evolution of results in each generation can be visually checked by an active BIM system. It is shown in the paper that GA can be used not only for schedule optimisation but also for developing project schedules from the scratch.	[122]
2014	PSP for reducing schedule overlaps.	The authors use a revised GA for minimizing schedule and workspace interference, as well as an active BIM-based simulation system for visualisation of the solution.	According to the authors, the developed system can be utilized as an active BIM tool that can be applied to sites with minimum special knowledge required from the administrator. Based on study results, the value of the schedule-workspace interference impact factor is reduced to 26.4% of the planned value (i.e. reduced by 73.6% compared to the plan). However, the proposed approach is used on a simple example and a detailed modification of the 4D model is considered important prior to its application on other projects.	[123]

Table 5. Active BIM applications for PSPs - extension

2014	PSP for generating a new schedule.	GA and matrix of constructability constraints (MoCC) are used for generating a new schedule.	Results indicate that the proposed approach can be used to produce completely constructible schedules. GA fitness function can be used to check constructability of the corresponding tasks, while the schedule-workspace interference can be confirmed visually by managers. Authors have concluded that the model can be extended by modifying 3D reader, by using correlation coefficients, by changing GA gene formulations, etc.	[124]
2015	PSP for generating a new schedule.	GA-based technique and MoCC for scheduling are used. Revit software is used for BIM model generation.	Results show that the proposed approach can be effectively used for generating construction schedule models in two degrees between simple and complex. The experiment was performed on a very simple BIM model and so the gained progress schedule only roughly corresponds to the global optimum.	[125]
2015	PSP with resource constraints.	The following procedures are used: evolutionary PSO algorithm; Revit application programming interface (API) in C# code; BIM-based information activities in MS Access database; MS Project-based scheduling by XML file parsing; simulations in Symphony.	Results indicate that the developed scheduling system can produce a schedule in case of construction time overrun and penalty payments. The obtained solution may not be a global optimum, because results include non-working days. The system has the following limitations: durations and productivity are based on manager's experience; weather limitations are not taken into consideration; simulation needs to be established manually, etc. These limitations may be considered in future research.	[126]
2015	Time-cost-quality trade-off problem.	GA is used for schedule optimisation. BIM model-based objects are transferred from BIM to data in the table via graphic converter. Structured query language is used for the calculation of quantities. The scheduling output is reported by Microsoft Project.	The study shows that the proposed model can demonstrate establishment of the quality balanced and cost optimal schedule generated by GA. The authors conclude that practical applicability of the model has to be tested, because factors affecting periods are different from actual characteristics.	[127]
2016	PSP for construction schedules as related to the time, cost and job-site movement optimisation.	Multi-objective GA, MoCC and 2D Pareto Fronts for generating a new schedule are used.	The results show the difference between the cost-driven and time-driven weight calculations via GA model. The cost objective score is increased in time-driven runs and decreased in cost-driven runs, as expected. The limitation of this approach lies in a large number of rounds required for GA calculations. As to future research, the authors consider that it would be appropriate to verify if better representation of objectives' relationships can be found through another multi-objective optimisation method. More 3D element types are needed to generate more realistic results.	[128]
2017	PSP for construction schedule optimisation	Simulated critical path method (CPM) is used to generate an initial schedule. Tabu-search algorithm is used for schedule optimisation.	Authors consider that the tabu-search algorithm can be used to determine an optimal construction project schedule. The weights of the objective function were set as different constants. Case study resulted in a 13% reduction in project duration, in 4% cost reduction, and in 49% decrease in resource fluctuation. In conclusion, the authors state that this method can successfully be used for improving schedules when multiple objectives are taken into consideration.	[129]

6. Discussion and conclusions

The aim of this paper was to provide a state-of-the-art review of solution methods and modelling tools, and to present their application in the optimisation of construction processes. For that purpose, an initial section provides a brief introduction into this subject, while the following two sections reveal solution methods and modelling tools that can nowadays be suitably used for optimisation. It was established that modern users have at their disposal a wide range of powerful optimisation algorithms, either heuristic or exact ones, and also several possibilities for optimisation problem modelling, such as via an algebraic modelling language, a spreadsheet, a specialized optimisation stand-alone software, an interactive computer language, or an internet application for optimisation.

Thereupon, the authors offer a short overview of optimisation problems related to construction processes, including their origins and definitions, and present successful applications, solution methods, and outcomes. After review of original

problems and applications, the authors state that in most cases modern applications of optimisation in construction processes are actually mere extensions or modifications of original formulations of optimisation problems, and that these problems are being solved using models with various combinations of exact, heuristic, meta- or hyper-heuristic methods. However, recent research reveals a clear trend of growing interest in the development of heuristic methods (such as GAs, PSO, ACO, etc.) for solving optimisation problems in construction processes. Various optimisation implementations in BIM are also shown in the final sections of the paper in order to highlight the potential of this field of research. In the review, it is shown that heuristic GAs are mostly used for solving BIM-oriented problems such as assignment problem, construction site layout problem, and project scheduling problem (which are currently the most frequently used problem-related applications in this approach). In the field of exact mathematical programming, there have been only some isolated attempts to combine MILP optimisation with BIM, and so there is still ample room for further development. Similar

conclusion can be made for applications based on bio-inspired optimisation approaches and BIM, as well as for mathematical optimisation of ERP systems. In this way, the literature gap on the aforesaid topics has been covered.

From the perspective of construction business management, it can be concluded that information systems can be successfully applied for processes like preparation, planning, monitoring, controlling, etc. Furthermore, these systems can also be adequately used after construction, e.g. for maintenance management, reconstructions, and demolition processes. However, the connection between information systems (like ERP and BIM) and optimisation currently seems to be a challenging task that needs to be addressed more intensively. An accelerated growth of various decision-making models, which combine optimisation approaches and information systems, offers a clear vision of future developments in this area. Moreover, it is

expected that connections between these two growing disciplines will also give rise to a new wave of active BIM developments aimed at automating processes in the field of construction project management. However, there still exists a number of NP-complete and NP-hard problems that cannot be solved to exact optimality in reasonable computational time by current software and hardware. Therefore, the anticipated progress in this field will in the future remain greatly dependent on available computer capacities.

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